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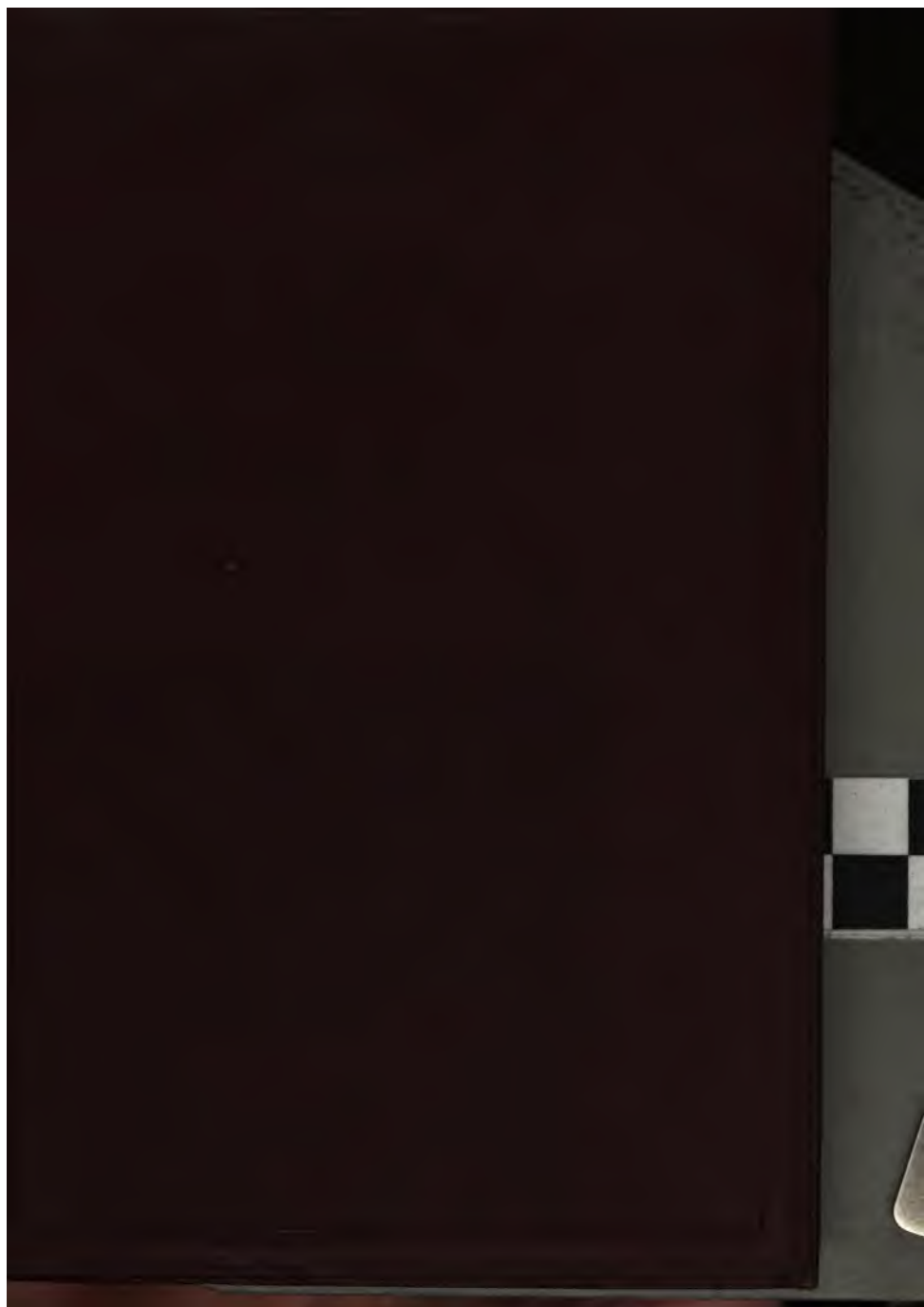
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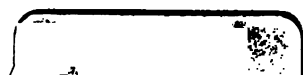
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BY
THE REV. T. DALTON, M.A.,
SENIOR MATHEMATICAL MASTER OF ETON COLLEGE.

London
MACMILLAN AND CO.
AND NEW YORK
1886

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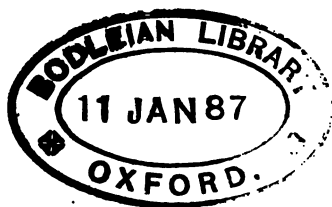


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KEY TO ALGEBRA.

PART I.

EXERCISE I.

1. $5+4+3-2-1+0=12-8=9.$
2. $5-4-3+2+1-0=8-7=1.$
3. $10+12+12+2+3=39.$
4. $25+24-6-8+1-0=50-14=36.$
5. $30+4-33-24+3+0=37-57=-20.$
6. $36-12+12-5+0=48-17=31.$
7. $8+15-0-9-12=23-21=2.$
8. $5+4-30-4+4-0=13-34=-21.$
9. $20+12+6+2+10=50.$
10. $8-3-4+15-0+0=16.$
11. $5+0-2-15-4-0=-16.$
12. $0+4+6-8-5-0=-3.$
13. $40+36-24-10+0=42.$
14. $45-40-0+48+48-3=98.$
15. $0-21+48-55+24-40=72-116=-44.$
16. $40-45+16-6-0+2=58-51=7.$
17. $240+120-36-140=360-176=184.$
18. $108-80+20-12-0=128-92=36.$
19. $30+48-0-24-0=78-24=54.$
20. $30-0-0+144-54=174-54=120.$
21. $16+60+0+10+18=104.$
22. $0-10+120+18-0=128.$
23. $24-0-45+0-48=-69.$
24. $0+50-16-0+40=74.$
25. $\frac{4}{4}+\frac{7}{7}-\frac{4}{4}=1+1-1=1.$
26. $\frac{8+0}{4}-\frac{6-1}{5}+\frac{20+0}{10}=2-\frac{5}{5}+2=3.$

27. $\frac{4+3+1}{4} - \frac{2+5-6}{1} + \frac{12-5-3}{2} = 2 - 1 + 2 = 3.$
28. $\frac{8}{1} - \frac{10}{4} + \frac{5}{2} + \frac{0}{5} = 8 - 2\frac{1}{2} + 2\frac{1}{2} + 0 = 8.$
29. $\frac{0+15}{5} - \frac{10+4}{7} - \frac{18}{3} = 3 - 2 - 6 = -5.$
30. $\frac{0+30-12}{8} - \frac{12+15-12}{1} + \frac{20+45}{8} = 2\frac{1}{4} - 15 + 8\frac{1}{8} = -4\frac{5}{8}.$
31. $16+4+0+25+9+1=55.$
32. $32+12-100+9+5=58-100=-42.$
33. $64-4+0+75+99=234.$
34. $80-28-0+60+54+9=203-28=175.$
35. $64+8+0+125+27+1=225.$
36. $256+48+16-40-14+750=1016.$
37. $125-18+3-16-36=58.$ 38. $256-0-250+108-1=113.$
39. $384+20+50+9+3=466.$ 40. $250-225+135+27-45=142.$
41. $32-48+0-600+30=-586.$ 42. $0-0+675-25-54=596.$
43. $16-80+300-500+625=361.$ 44. $162-108+72-12+1=115.$
45. $160-80+200-75+45-15=235.$
46. $0+320+96-450-270+75=-229.$
47. $128+32-20+0-180+12=-28.$
48. $2560-2520+2160-900+0=1300.$
49. $\frac{32+0}{2} - \frac{24-36}{1} + \frac{7+30}{4} = 16+12+9\frac{1}{4}=37\frac{1}{4}.$
50. $\frac{24+0}{4} - \frac{30-9}{5} + \frac{16-8+4}{3} = 6-4\frac{1}{5}+4=5\frac{4}{5}.$
51. $\frac{50+75-9}{20} + \frac{48-0+0}{6} + \frac{9-12+5}{2} = 5\frac{1}{4}+8+1=14\frac{1}{4}.$
52. $\frac{125+225+180-135}{50} - \frac{160-180}{25} + \frac{12-54+108}{4} = \frac{395}{50} + \frac{20}{25} + \frac{66}{4}$
 $= 7\frac{9}{10} + \frac{4}{5} + 16\frac{1}{2} = 25\frac{1}{2}.$
53. $\frac{128-192+64}{15} - \frac{81-18+6}{60} + \frac{125-125+15-10}{4} = 0 - 1\frac{1}{10} + 1\frac{1}{4} = \frac{1}{10}.$
54. $\frac{4+6+5}{20} - \frac{270-900+625}{3} + \frac{16-100+54}{15} = \frac{3}{4} + 1\frac{1}{3} - 2 = \frac{5}{12}.$

EXERCISE II.

3

55. $4^3 + 3 \times 5^3 - 0^3 + 4 \times 1^3 = 16 + 250 - 0 + 4 = 270.$

56. $\frac{24-12}{18-12} - \frac{72-0}{48-0} + \frac{75-256}{75-80} = 2 - 1\frac{1}{2} + 36\frac{1}{5} = 36\frac{7}{10}.$

EXERCISE II.

2. $16a - 16b - 23.$ 4. $17x + 28y - 3z.$ 6. $12a^2 - 11a - 3.$

8. $18x^2 + y^2 - 3xy + 5x + 17y.$ 10. $22x^5 - 4x^4 + x + 10.$

12. $20x^4 + 16x^3 + 9x^2 + 5x + 8.$ 14. $-6a + 10b + 7c - 15d.$

15.

$$\begin{array}{r} -6a^3 + 12b^2 + c^3 + 2bc - 7ca + 2ab \\ a^3 + 5c^2 - 8bc + 8ab \\ 4b^3 - 2c^2 - 3ca - 10ab \\ \hline -5a^3 + 16b^2 + 4c^2 - bc - 10ca \end{array}$$

16.

$$\begin{array}{r} 6x^3 + 7y^2 + 2z^2 - 5yz + 3xz \\ 2x^3 - 3z^2 + 4yz - 2xz \\ 7x^3 + 4y^2 - 3yz + 4xz \\ - 2x^2 + 13yz \\ \hline 13x^3 + 11y^2 - z^2 + 9yz + 5xz \end{array}$$

17.

$$\begin{array}{r} 7x^2 - 3z^2 + 2yz + 12xz - 4xy \\ 9x^2 + 2y^2 + 4xz + 2xy \\ x^2 - y^2 - 3xz \\ - 4x^2 + 12y^2 - 13xz + 12xy \\ \hline 13x^2 + 13y^2 - 3z^2 + 2yz + 10xy \end{array}$$

18.

$$\begin{array}{r} 4a^2 - 2c^2 - 3b - 14c \\ -2a^2 + 3b^2 - 4c^2 + 7b + 11c \\ a^2 - 2b^2 + 11c^2 + 8a - 9b \\ 3a^2 - 9a + 4b + 2c \\ \hline 6a^2 + b^2 + 5c^2 - a - b - c \end{array}$$

19.

$$\begin{array}{r} -15a^3 - a^2b - 2b^3 \\ -4a^3 - 11a^2b - 7ab^2 + 9b^3 \\ 6a^3 - 4ab^2 + 9b^3 \\ -2a^3 + 6a^2b - 2a^3 + 6a^2b - 11ab^2 + 15b^3 \\ \hline -15a^3 + 6a^2b - 11ab^2 + 15b^3 \end{array}$$

20.

$$\begin{array}{r} 4a^2 + 12bc \\ 6b^2 - 7c^2 + 9bc + 2ca \\ 3a^2 + 2b^2 + bc - 3ca - 7ab \\ -8b^2 - c^2 - 10bc \\ \hline 7a^2 - 8c^2 + 12bc - ca - 7ab \end{array}$$

21.

$$\begin{array}{r} 3x^3 - 10y^3 + 5z^2 - 7yz \\ - x^3 + 4y^2 - 10z^2 + 3xy \\ z^2 + 11yz + 8xz - 2xy \\ 4x^2 - 4yz + xz \\ - 2x^3 + 6y^2 - 9xz - xy \\ \hline \end{array}$$

22.

$$\begin{array}{r} 2a^3 - 7a^2b + 4b^3 \\ - a^3 + 6ab^2 + 2c^3 \\ 5a^2b - 2ab^2 + b^3 - c^3 \\ - 4ab^3 - 7c^3 + 2b^2c \\ \hline a^3 - 2a^2b + 5b^4 - 6c^3 + 2b^2c \end{array}$$

23.

$$\begin{array}{r} 1 + 3x + 2x^3 - x^5 \\ 4 - 2x^2 - 8x^3 + 7x^4 \\ 12x - 4x^3 + 12x^5 \\ 5x^3 + 7x^3 - 11x^5 \\ \hline 5 + 15x - x^2 + x^3 + 7x^4 \end{array}$$

EXERCISE III.

24.
$$\begin{array}{r} 7x^3y + 8xy^3 - 6y^4 \\ - 8x^4 + 2x^2y + 2x^2y^2 - 4xy^3 + 5y^4 \\ 3x^4 - 4x^2y - 2x^2y^2 + 3xy^3 + 3y^4 \\ \hline - 4x^4 + 5x^2y + 12x^2y^2 + 7xy^3 \end{array}$$
25.
$$\begin{array}{r} 11x^4 - 2xy^3 - 8y^4 \\ 3x^3y - 2x^2y^2 + 7xy^3 - 8y^4 \\ 8x^4 - 7x^2y - 12x^4 + 4x^2y + 5x^2y^2 - 3x^2y^2 - 5xy^3 + 7y^4 \\ \hline 7x^4 \dots\dots\dots \end{array}$$
26.
$$\begin{array}{r} 4a^5 + 12a^3 - a - 10 \\ - 3a^5 - 6a^4 - a^3 + 2a^2 - 7 \\ - a^5 - 2a^4 + 9a^2 + 4a \\ - a^4 + 4a^3 \quad 11a + 9 \\ \hline 3a^4 + 15a^3 + 11a^2 + 14a - 13 \end{array}$$
27.
$$\begin{array}{r} 11x^3 + 14x^2y - 7xy^2 + z^3 \\ x^3 - 2x^2y + 8xy^2 - 7y^3 + 3y^2z - 2z^3 \\ 4x^3 + 2y^3 - 11z^3 + 4yz^2 - 3xyz \\ + 4xy^2 + 8y^3 - 7y^2z + 12z^3 - 4yz^2 + 4xyz \\ \hline 16x^3 + 12x^2y - 2y^3 - 4y^2z + xyz \end{array}$$
28.
$$\begin{array}{r} a^3 - 11a^2b - 4ab^3 - 2b^3 - bc^3 \\ 9a^2b + ab^3 + 4b^3 - abc + 6c^3 - ac^3 \\ - a^3 + 3ab^3 - 2b^3 + 4bc^3 - c^3 \\ 2a^2b - 3bc^2 + 3abc - 7c^3 \\ \hline \dots\dots\dots 2abc - ac^3 \end{array}$$
29.
$$\begin{array}{r} a^3x + 2a^2x^2 - 7x^4 \\ 5a^4 - 3a^2x^3 + 4ax^3 - x^4 \\ - 2a^4 + 5a^3x + 3x^4 - 2x^3 - 3 \\ - a^4 - 6a^3x - 2x^4 + 4x^3 - 9 \\ - a^2x^3 + 7x^4 - 2x^3 + x^2 - 1 \\ \hline 2a^4 - 2a^2x^3 + 4ax^3 + x^2 - 13 \end{array}$$
30.
$$\begin{array}{r} - x^3z + x^2yz - 2xz^3 - 7z^4 \\ - x^4 - 3xz^3 + 3y^4 - 2x^2y^2 - 4xy^3 \\ - 2x^4 + 4xz^3 - y^4 - 10xy^2z + 4x^2z^2 \\ 3x^4 - x^2yz + 7z^4 + 3y^4 - 4x^2z^2 + 8xyz^2 \\ 4xz^3 + 2y^4 + 2x^2y^2 + 4xy^3 + 10xy^2z \\ - 2xz^3 - 7y^4 - 8xyz^2 \\ \hline \dots\dots\dots \end{array}$$

EXERCISE III.

2. $14a + 2b + c + 8.$ 4. $x^2 + 2xy + 11y^2 + 5.$
6. $8a + 21b - 21c + 11.$ 8. $6a^2 - 9b^2 + 12c^2 + 2d^2.$
10. $-10a^3 + 4a^2b - b^3.$ 12. $2x^3 + 24xy + 2y^3.$
14. $2a + 6b - 4c - 2.$ 16. $a^2 - 10b^2 - 8c^2 + 2bc + 6ca - 2ab.$
18. $-13a^3 + 3b^3 - 3c^3 + 16bc - 4ab.$ 20. $-6x^3 + 27y^3 - 3z^3 + 4yz^2 - 11z^2x,$

EXERCISE III.

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$$\begin{array}{r} 21. \quad 8a^3 + 3a^2 - 2a + 7 \\ \quad \quad 3a^3 \quad \quad + 7a - 4 \\ \hline \quad \quad 5a^3 + 3a^2 - 9a + 11 \end{array}$$

22.
$$\begin{array}{r} 11x^4 - 2x^3 + 3x^2 - 8x \\ 3x^4 \quad \quad - x^2 + 7x - 14 \\ \hline 8x^4 - 2x^3 + 4x^2 - 15x + 14 \end{array}$$

23.
$$\frac{4x^3 - 2xy^2}{2x^3 - 2x^2y + xy^2 + y^3} \div \frac{2x^3 + 2x^2y - 3xy^2 - y^3}{2x^3 - 2x^2y + xy^2 + y^3}$$

$$\begin{array}{r} 24. \quad 12a^5 - 7a^4 + 3a^3 \qquad - 7 \\ \quad \quad 2a^4 \qquad - 3a^2 + 2a - 5 \\ \hline 12a^5 - 9a^4 + 3a^3 + 3a^2 - 2a - 2 \end{array}$$

$$25. \frac{2x^5 - \frac{12x^4y}{x^4y + 7x^2y^2} + \frac{3x^2y^3 + 2xy^4 - y^5}{+ 2xy^4 + y^5}}{-2x^5 + 13x^4y - 7x^2y^3 + 3x^2y^5 - 2y^5}$$

$$\begin{array}{r} 26. \quad \begin{array}{r} 4a^3b + 7a^2b^2 - 2b^4 \\ 2a^4 \quad - \quad 7a^2b^3 - 2ab^3 + 3b^4 \\ \hline -2a^4 + 4a^3b + 14a^2b^3 + 2ab^3 - 5b^4 \end{array} \qquad 27. \quad \begin{array}{r} x^3 \quad \quad \quad + 64 \\ x^3 - 4x^2 + 16x \\ \hline \quad \quad \quad 4x^2 - 16x + 64 \end{array} \end{array}$$

$$\begin{array}{r} 28. \quad x^5 + 3xy^4 \\ x^5 + 2x^4y + 3x^3y^2 - 2xy^4 + y^5 \\ \hline -2x^4y - 3x^3y^2 + 5xy^4 - y^5 \end{array}$$

$$\begin{array}{r} 29. \quad 16x^4 \\ \underline{8x^3y + 4x^2y^2 + 2xy^3 + y^4} \\ 16x^4 - 8x^3y - 4x^2y^2 - 2xy^3 \end{array}$$

$$\begin{array}{r} 30. \quad \begin{array}{r} 8x^2y + 4x^2y^2 + 2xy^3 + y^4 \\ 16x^4 \end{array} \\ \hline -16x^4 + 8x^2y + 4x^2y^2 + 2xy^3 \end{array}$$

31. $\frac{4a^3}{3a^3 + 2a^2b + 6ab^2 - 2b^3}$
 $\frac{a^3 - 2a^2b - 6ab^2 + 2b^3}{3a^3 + 2a^2b + 6ab^2 - 2b^3}$

32.
$$\begin{array}{r} 7xy^3 - 2x^2y^2 \\ -4x^4 + 3x^2y^2 - 2xy^3 + y^4 \\ \hline 4x^4 + 7xy^3 - 5x^2y^2 + 2xy^3 - y^4 \end{array}$$

83.
$$\begin{array}{r} 13a^5 + 6a^4b - 2a^3b^2 \\ -13a^5 - 6a^4b + 2a^3b^2 + 7ab^4 - 2b^5 \\ \hline \end{array}$$

34.
$$\frac{4x^4 - 3x^3y + xy^3}{4x^4 + 2x^2y^2 - xy^3 - y^4} - \frac{3x^3y - 2x^2y^2 + 2xy^3 + y^4}{4x^4 + 2x^2y^2 - xy^3 - y^4}$$

$$35. \quad \begin{array}{r} 2x^3 + 4x^2 \quad + 9 \\ 4x^3 - x^2 + 4x - 2 \\ \hline 6x^3 + 3x^2 + 4x + 7 \end{array} \quad \begin{array}{r} 6x^3 \quad + 4x + 7 \\ 6x^3 + 3x^2 + 4x + 7 \\ \hline -3x^2 \end{array}$$

36.
$$\begin{array}{r} 4x^4 - 2x^3 + 3x^2 + 9 \\ 6x^5 + 2x^4 + 2x^3 + 5x - 3 \\ \hline 6x^5 + 6x^4 + 3x^3 + 5x + 6 \end{array}$$

$$\begin{array}{r} 12x^5 + 5x \\ 6x^5 + 6x^4 + 3x^2 + 5x + 6 \\ \hline 6x^5 - 6x^4 - 3x^2 - 6 \end{array}$$

$$\begin{array}{r} 37. \quad 6a^3 + 4a^2x + 3ax^2 \\ \quad \quad 2a^2x - ax^2 + x^3 \\ \quad \quad \quad a^2x - 2ax^2 + 3x^3 \\ \hline 6a^3 + 7a^2x \qquad \qquad + 4x^3 \end{array}$$

$$\begin{array}{r} 6a^3 + 2a^2x - 4x^3 \\ 6a^3 + 7a^2x + 4x^3 \\ \hline -5a^2x - 8x^3 \end{array}$$

$$\begin{array}{r} 38. \quad a^4 - 2a^3x \qquad \qquad \qquad + x^4 \\ \quad 2a^4 + 5a^3x + 3a^2x^2 \qquad \quad - 5x^4 \\ \quad 6a^4 + a^3x + a^2x^2 - 4ax^3 \\ \hline \quad 9a^4 + 4a^3x + 4a^2x^2 - 4ax^3 - 4x^4 \end{array}$$

$$\frac{10a^4 - 4a^3x - 2a^2x^2 - ax^3}{9a^4 + 4a^3x + 4a^2x^2 - 4ax^3 - 4x^4}$$

39.
$$\begin{array}{r} 2x^3 + 9x \\ 5x^3 + 3x^2 \\ \hline 7x^3 + 3x^2 + 9x \\ 7x^3 + 3x^2 - 5x + 7 \\ \hline 14x - 7 \end{array}$$

$$\begin{array}{r} x^3 + 3x - 2 \\ 2x^3 + x^2 - x + 5 \\ 4x^3 + 2x^2 - 7x + 4 \\ \hline 7x^3 + 3x^2 - 5x + 7 \end{array}$$

$$\begin{array}{r}
 40. \quad \begin{array}{r} 6x^5 \\ x^5 - 2x^4y + x^3y^2 \\ 6x^5 + 4x^4y - 2x^3y^2 \\ \hline 13x^5 + 2x^4y - x^3y^2 + 2x^2y^3 + 4y^5 \end{array} \\
 \end{array}
 \quad
 \begin{array}{r}
 12x^5 \\ 2x^5 - 4x^4y \\ 6x^4y \\ \hline 14x^5 + 2x^4y \\ 18x^5 + 2x^4y - x^3y^2 + 2x^2y^3 \\ \hline x^5
 \end{array}$$

EXERCISE IV.

$$\begin{array}{ll}
 31. \quad \begin{array}{r} 2a + b \\ 2a + 3b \\ \hline 4a^2 + 2ab \\ + 6ab + 3b^2 \\ \hline 4a^2 + 8ab + 3b^2 \end{array} & 32. \quad \begin{array}{r} 3x + 2y \\ 4x + 3y \\ \hline 12x^2 + 8xy \\ 9xy + 6y^2 \\ \hline 12x^2 + 17xy + 6y^2 \end{array} \\
 33. \quad \begin{array}{r} 5y - 7z \\ 4y - 2z \\ \hline 20y^2 - 28yz \\ - 10yz + 14z^2 \\ \hline 20y^2 - 88yz + 14z^2 \end{array} & 34. \quad \begin{array}{r} a + b \\ a - b \\ \hline a^2 + ab \\ - ab - b^2 \\ \hline a^2 - b^2 \end{array} \\
 35. \quad \begin{array}{r} x^3 + y^3 \\ x^2 - y^2 \\ \hline x^4 + x^2y^2 \\ - x^2y^2 - y^4 \\ \hline x^4 - y^4 \end{array} & 36. \quad \begin{array}{r} 3a^3 - 4a + 5 \\ 2a + 5 \\ \hline 6a^3 - 8a^2 + 10a \\ 15a^2 - 20a + 25 \\ \hline 6a^3 + 7a^2 - 10a + 25 \end{array} \\
 37. \quad \begin{array}{r} 5x^2 + 4x - 3 \\ 3x + 4 \\ \hline 15x^3 + 12x^2 - 9x \\ 20x^2 + 16x - 12 \\ \hline 15x^3 + 32x^2 + 7x - 12 \end{array} & 38. \quad \begin{array}{r} 2x^2 - 3xy + y^2 \\ 2x + 3y \\ \hline 4x^3 - 6x^2y + 2xy^2 \\ 6x^2y - 9xy^2 + 3y^3 \\ \hline 4x^3 - 7xy^2 + 3y^3 \end{array} \\
 39. \quad \begin{array}{r} 4x^3 - 2ax + a^3 \\ 2x + a \\ \hline 8x^3 - 4ax^2 + 2a^2x \\ 4ax^2 - 2a^2x + a^3 \\ \hline 8x^3 + a^3 \end{array} & 40. \quad \begin{array}{r} 7a^2 - 5a + 3 \\ 3a - 2 \\ \hline 21a^3 - 15a^2 + 9a \\ - 14a^2 + 10a - 6 \\ \hline 21a^3 - 29a^2 + 19a - 6 \end{array} \\
 41. \quad \begin{array}{r} 3x^2 - 7x + 1 \\ 2x - 5 \\ \hline 6x^3 - 14x^2 + 2x \\ - 15x^2 + 35x - 5 \\ \hline 6x^3 - 29x^2 + 37x - 5 \end{array} & 42. \quad \begin{array}{r} x^3 + ax + a^2 \\ x - a \\ \hline x^3 + ax^2 + a^2x \\ - ax^2 - a^2x - a^3 \\ \hline x^3 - a^3 \end{array} \\
 43. \quad \begin{array}{r} 2x^2 - 4bx + 3b^2 \\ 2x - 3b \\ \hline 4x^3 - 8bx^2 + 6b^2x \\ - 6bx^2 + 12b^2x - 9b^3 \\ \hline 4x^3 - 14bx^2 + 18b^2x - 9b^3 \end{array} & 44. \quad \begin{array}{r} 4x^2 - 3x^2 - 2x + 5 \\ 3x - 5 \\ \hline 12x^4 - 9x^3 - 6x^2 + 15x \\ - 20x^3 + 15x^2 + 10x - 25 \\ \hline 12x^4 - 29x^3 + 9x^2 + 25x - 25 \end{array}
 \end{array}$$

EXERCISE IV.

7

45.
$$\begin{array}{r} 3a^3 + 5a^2 - 7a + 3 \\ 2a - 8 \\ \hline 6a^4 + 10a^3 - 14a^2 + 6a \\ - 9a^3 - 15a^2 + 21a - 9 \\ \hline 6a^4 + a^3 - 29a^2 + 27a - 9 \end{array}$$
46.
$$\begin{array}{r} 5a^3 - 3a^2b - 5ab^2 + 4b^3 \\ 3a + 5b \\ \hline 15a^4 - 9a^3b - 15a^2b^2 + 12ab^3 \\ 25a^3b - 15a^2b^2 - 25ab^3 + 20b^4 \\ \hline 15a^4 + 16a^3b - 30a^2b^2 - 13ab^3 + 20b^4 \end{array}$$
47.
$$\begin{array}{r} 2y^3 + 3by^2 - 2b^2y - 3b^3 \\ 4y - 8b \\ \hline 8y^4 + 12by^3 - 8b^2y^2 - 12b^3y \\ - 6by^3 - 9b^2y^2 + 6b^3y + 9b^4 \\ \hline 8y^4 + 6by^3 - 17b^2y^2 - 6b^3y + 9b^4 \end{array}$$
48.
$$\begin{array}{r} x^4 - ax^3 + a^2x^2 - a^3x + a^4 \\ x + a \\ \hline x^5 - ax^4 + a^2x^3 - a^3x^2 + a^4x \\ ax^4 - a^2x^3 + a^3x^2 - a^4x + a^5 \\ \hline x^5 + \dots + a^5 \end{array}$$
49.
$$\begin{array}{r} 3x^4 - 2x^3y + 4x^2y^2 - 7xy^3 + 16y^4 \\ 2x - 3y \\ \hline 6x^5 - 4x^4y + 8x^3y^2 - 14x^2y^3 + 32xy^4 \\ - 9x^4y + 6x^3y^2 - 12x^2y^3 + 21xy^4 - 48y^5 \\ \hline 6x^5 - 13x^4y + 14x^3y^2 - 26x^2y^3 + 53xy^4 - 48y^5 \end{array}$$
50.
$$\begin{array}{r} a^2b^2 + ab + 1 \\ ab - 1 \\ \hline a^3b^3 + a^2b^2 + ab \\ - a^3b^3 - ab - 1 \\ \hline a^3b^3 - 1 \end{array}$$
51.
$$\begin{array}{r} 2x^3 - 8x + 4 \\ 8x^2 - 2x - 5 \\ \hline 6x^4 - 9x^3 + 12x^2 \\ - 4x^3 + 6x^2 - 8x \\ - 10x^2 + 15x - 20 \\ \hline 6x^4 - 13x^3 + 8x^2 + 7x - 20 \end{array}$$
52.
$$\begin{array}{r} 4x^3 + 2x + 3 \\ x^2 - 3x - 4 \\ \hline 4x^4 + 2x^3 + 8x^2 \\ - 12x^3 - 6x^2 - 9x \\ - 16x^2 - 8x - 12 \\ \hline 4x^4 - 10x^3 - 19x^2 - 17x - 12 \end{array}$$
53.
$$\begin{array}{r} 8a^3 - 3a - 1 \\ 2a^2 + 4a + 3 \\ \hline 6a^4 - 6a^3 - 2a^2 \\ 12a^3 - 12a^2 - 4a \\ 9a^3 - 9a - 3 \\ \hline 6a^4 + 6a^3 - 5a^2 - 13a - 3 \end{array}$$
54.
$$\begin{array}{r} 7y^3 - 4y + 3 \\ 2y^2 - 4y - 3 \\ \hline 14y^4 - 8y^3 + 6y^2 \\ - 28y^3 + 16y^2 - 12y \\ - 21y^2 + 12y - 9 \\ \hline 14y^4 - 36y^3 + y^2 - 9 \end{array}$$
55.
$$\begin{array}{r} y^3 + 4y + 16 \\ y^2 - 4y + 16 \\ \hline y^4 + 4y^3 + 16y^2 \\ - 4y^3 - 16y^2 - 64y \\ 16y^2 + 64y + 256 \\ \hline y^4 + 16y^2 + 256 \end{array}$$

56. $b^4 - 2b^2 + 1$

$$\frac{b^4 + 2b^2 + 1}{b^4 - 2b^2 + 1}$$

$$\frac{b^4 - 2b^2 + 1}{2b^4 - 4b^2 + 2b^2}$$

$$\frac{b^4 - 2b^2 + 1}{b^4 - 2b^2 + 1}$$

$$\frac{b^4 - 2b^2 + 1}{b^4 - 2b^2 + 1}$$

58. $x^6 - 4x^3 + 4$

$$\frac{x^6 + 4x^3 + 4}{x^6 - 4x^3 + 4}$$

$$\frac{x^6 - 4x^3 + 4}{4x^3 - 16x^3 + 16x^3}$$

$$\frac{4x^3 - 16x^3 + 16x^3}{4x^3 - 16x^3 + 16}$$

$$\frac{x^{12} - 8x^6 + 16}{x^{12} - 8x^6 + 16}$$

60.

$$2a^2 - 3ab + 3b^2$$

$$3a^2 - 4ab - 2b^2$$

$$6a^4 - 9a^3b + 9a^2b^2$$

$$- 8a^3b + 12a^2b^2 - 12ab^3$$

$$- 4a^2b^3 + 6ab^3 - 6b^4$$

$$6a^4 - 17a^3b + 17a^2b^2 - 6ab^3 - 6b^4$$

61.

$$4x^3 - 8ax + a^2$$

$$2x^3 - 4ax - 5a^2$$

$$8x^4 - 6ax^3 + 2a^2x^2$$

$$- 16ax^3 + 12a^2x^2 - 4a^2x$$

$$- 20a^2x^2 + 15a^3x - 5a^4$$

$$8x^4 - 22ax^3 - 6a^2x^2 + 11a^3x - 5a^4$$

62.

$$5x^2 + 4xy - 2y^2$$

$$2x^2 - 3xy - 4y^2$$

$$10x^4 + 8x^2y - 4x^2y^2$$

$$- 15x^2y - 12x^2y^2 + 6xy^3$$

$$- 20x^2y^2 - 16xy^3 + 8y^4$$

$$10x^4 - 7x^2y - 36x^2y^2 - 10xy^3 + 8y^4$$

63.

$$2y^4 - 3b^2y^2 + b^4$$

$$3y^4 - 2b^2y^2 - 2b^4$$

$$6y^6 - 9b^2y^4 + 3b^4y^4$$

$$- 4b^2y^4 + 6b^4y^4 - 2b^6y^2$$

$$- 4b^4y^4 + 6b^6y^2 - 2b^8$$

$$6y^6 - 13b^2y^4 + 5b^4y^4 + 4b^6y^2 - 2b^8$$

64.

$$x^5 - x^2y + xy^2 - y^3$$

$$x^3 + 2xy + y^3$$

$$x^5 - x^4y + x^3y^2 - x^2y^3$$

$$2x^4y - 2x^3y^2 + 2x^2y^3 - 2xy^4$$

$$x^3y^2 - x^2y^3 + xy^4 - y^5$$

$$x^5 + x^4y - xy^4 - y^5$$

57. $a^4 + 3a^2 - 2$

$$\frac{a^4 - 4a^2 + 1}{a^4 + 3a^2 - 2}$$

$$\frac{a^4 + 3a^2 - 2}{a^4 - 4a^2 - 12a^4 + 8a^2}$$

$$\frac{a^4 + 3a^2 - 2}{a^4 - 4a^2 - 12a^4 + 8a^2}$$

$$\frac{a^4 - 4a^2 - 12a^4 + 8a^2}{a^4 - 4a^2 - 12a^4 + 8a^2}$$

59. $2y^6 + 3y^3 + 5$

$$\frac{3y^6 - 2y^3 - 3}{2y^6 + 3y^3 + 5}$$

$$\frac{6y^{12} + 9y^9 + 15y^6}{3y^6 - 2y^3 - 3}$$

$$- 4y^3 - 6y^6 - 10y^3 - 15$$

$$- 6y^6 - 9y^3 - 15$$

$$6y^{12} + 5y^9 + 3y^6 - 19y^3 - 15$$

EXERCISE IV.

9

65.
$$\begin{array}{r} a^3 + 2a^2b + 4ab^2 + 8b^3 \\ a^3 - 4ab + 4b^3 \\ \hline a^3 + 2a^2b + 4a^2b^2 + 8a^2b^3 \\ - 4a^4b - 8a^3b^2 - 16a^2b^3 - 32ab^4 \\ \hline 4a^2b^2 + 8a^2b^3 + 16ab^4 + 32b^5 \\ a^3 - 2a^4b \quad - 16ab^4 + 32b^5 \end{array}$$
66.
$$\begin{array}{r} 2x^4 - 3x + 4 \\ 2x - 8 \\ \hline 4x^5 - 6x^4 - 6x^3 + 8x \\ - 6x^4 + 9x - 12 \\ \hline 4x^5 - 6x^4 - 6x^3 + 17x - 12 \end{array}$$
67.
$$\begin{array}{r} 3a^5 - 2a^3 + 3a - 1 \\ 3a + 2 \\ \hline 9a^6 - 6a^4 - 4a^3 + 9a^2 - 3a \\ 6a^5 - 4a^2 + 6a - 2 \\ \hline 9a^6 + 6a^5 - 6a^4 - 4a^3 + 9a^2 + 3a - 2 \end{array}$$
68.
$$\begin{array}{r} 2x^4 - 3xy^3 + y^4 \\ 3x - y \\ \hline 6x^5 - 9x^2y^3 + 3xy^4 \\ - 2x^4y + 3xy^4 - y^5 \\ \hline 6x^5 - 2x^4y - 9x^2y^3 + 6xy^4 - y^5 \end{array}$$
69.
$$\begin{array}{r} x^5 - 3ax^4 + 2a^5 \\ 2x + 3a \\ \hline 2x^6 - 6ax^5 + 4a^5x \\ 3ax^5 - 9a^2x^4 + 6a^6 \\ \hline 2x^6 - 3ax^5 - 9a^2x^4 + 4a^5x + 6a^6 \end{array}$$
70.
$$\begin{array}{r} x^5 - 2x^3 + 3 \\ 2x - 5 \\ \hline 2x^6 - 4x^4 + 6x \\ - 5x^5 + 10x^3 - 15 \\ \hline 2x^6 - 5x^5 - 4x^4 + 10x^3 + 6x - 15 \end{array}$$
71.
$$\begin{array}{r} x^5 - a^2x^4 + a^4x^3 - 2a^6 \\ 2x - 3a \\ \hline 2x^7 - 2a^2x^5 + 2a^4x^3 - 4a^6x \\ - 3ax^6 + 3a^3x^4 - 3a^5x^2 + 6a^7 \\ \hline 2x^7 - 3ax^6 - 2a^2x^5 + 3a^3x^4 + 2a^4x^3 - 3a^5x^2 - 4a^6x + 6a^7 \end{array}$$
72.
$$\begin{array}{r} x^3 - 2x + 1 \\ x^2 - 4 \\ \hline x^5 - 2x^3 + x^2 \\ - 4x^3 + 8x - 4 \\ \hline x^5 - 6x^3 + x^2 + 8x - 4 \end{array}$$
73.
$$\begin{array}{r} x^3 - 2x^2 + 3 \\ 2x^3 - 3x - 1 \\ 2x^7 - 4x^4 + 6x^3 - 9x \\ - 3x^6 - x^5 + 2x^2 - 8 \\ \hline 2x^7 - 3x^6 - x^5 - 4x^4 + 6x^3 + 8x^2 - 9x - 3 \end{array}$$
74.
$$\begin{array}{r} a^5 - 2a^2b^3 - b^5 \\ a^3 - ab^3 + b^3 \\ \hline a^5 - 2a^2b^3 - a^3b^5 + ab^7 \\ - a^6b^3 + 2a^4b^4 - 2a^2b^5 - b^6 \\ \hline a^5 - 3a^2b^3 + a^3b^5 + 2a^4b^4 - 3a^2b^5 + ab^7 - b^6 \end{array}$$

75.

$$\begin{array}{r}
 a^6 - a^2bc + b^2c^2 \\
 a^4 - bc \\
 \hline
 a^{10} - a^6bc + a^4b^2c^2 \\
 \hline
 a^{10} - 2a^6bc + a^4b^2c^2 + a^2b^2c^2 - b^2c^3
 \end{array}$$

76.

$$\begin{array}{r}
 a^2 + b^2 + c^2 - ab - ac - bc \\
 a + b + c \\
 \hline
 a^3 + ab^2 + ac^2 - a^2b - a^2c - abc \\
 a^2b + b^3 + bc^2 - ab^2 - abc - b^2c \\
 a^3c + b^2c + c^3 - abc - ac^2 - bc^2 \\
 \hline
 a^3 + b^3 + c^3 - 3abc
 \end{array}$$

77.

$$\begin{array}{r}
 a^2 + b^3 + c^2 + ab + ac - bc \\
 a - b - c \\
 \hline
 a^3 + ab^2 + ac^2 + a^2b + a^2c - abc \\
 - a^2b - b^3 - bc^2 - ab^2 - abc + b^2c \\
 - a^2c - b^2c - c^3 - abc - ac^2 + bc^2 \\
 \hline
 a^3 - b^3 - c^3 - 3abc
 \end{array}$$

78.

$$\begin{array}{r}
 x^3 + 4y^2 + 3z^2 \\
 x^2 - 2y^2 - 3z^2 \\
 \hline
 x^4 + 4x^2y^2 + 8x^2z^2 \\
 - 2x^2y^2 - 8y^4 - 6y^2z^2 \\
 - 3x^2z^2 - 12y^2z^2 - 9z^4 \\
 \hline
 x^4 + 2x^2y^2 - 8y^4 - 18y^2z^2 - 9z^4
 \end{array}$$

79.

$$\begin{array}{r}
 a^2 + b^2 + 9 - 3a + 3b + ab \\
 a - b + 3 \\
 \hline
 a^3 + ab^2 + 9a - 3a^2 + 3ab + a^2b \\
 - a^2b - b^3 - 9b + 3ab - 3b^2 - ab^2 \\
 3a^2 + 3b^2 + 27 - 9a + 9b + 3ab \\
 \hline
 a^3 - b^3 + 9ab + 27
 \end{array}$$

80.

$$\begin{array}{r}
 4x^2 + 9y^2 + z^2 - 6xy - 2xz - 3yz \\
 2x + 3y + z \\
 \hline
 8x^3 + 18xy^2 + 2xz^2 - 12x^2y - 4x^2z - 6xyz \\
 12x^2y + 27y^3 + 3yz^2 - 18xy^2 - 6xyz - 9y^2z \\
 4x^2z + 9y^2z + z^3 - 6xyz - 2xz^2 - 3yz^2 \\
 \hline
 8x^3 + 27y^3 + z^3 - 18xyz
 \end{array}$$

81.

$$\begin{array}{r}
 a + b \\
 a + b \\
 \hline
 a^2 + ab \\
 ab + b^2 \\
 \hline
 a^2 + 2ab + b^2
 \end{array}$$

83.

$$\begin{array}{r}
 3x - 4a \\
 3x - 4a \\
 \hline
 9x^2 - 12ax \\
 - 12ax + 16a^2 \\
 \hline
 9x^2 - 24ax + 16a^2
 \end{array}$$

82.

$$\begin{array}{r}
 2x + 3y \\
 2x + 3y \\
 \hline
 4x^2 + 6xy \\
 6xy + 9y^2 \\
 \hline
 4x^2 + 12xy + 9y^2
 \end{array}$$

84.

$$\begin{array}{r}
 4y - 5b \\
 4y - 5b \\
 \hline
 16y^2 - 20by \\
 - 20by + 25b^2 \\
 \hline
 16y^2 - 40by + 25b^2
 \end{array}$$

$$\begin{array}{r}
 85. \quad a + b + c \\
 a + b + c \\
 \hline
 a^2 + ab + ac \\
 \quad ab + b^2 + bc \\
 \quad \quad ac + bc + c^2 \\
 \hline
 a^2 + 2ab + 2ac + b^2 + 2bc + c^2
 \end{array}$$

$$\begin{array}{r}
 86. \quad x^2 - 3x + 4 \\
 x^2 - 3x + 4 \\
 \hline
 x^4 - 3x^3 + 4x^2 \\
 \quad - 3x^3 + 9x^2 - 12x \\
 \quad \quad 4x^2 - 12x + 16 \\
 \hline
 x^4 - 6x^3 + 17x^2 - 24x + 16
 \end{array}$$

$$\begin{array}{r}
 87. \quad a + b \\
 a + b \\
 \hline
 a^2 + ab \\
 \quad ab + b^2 \\
 \hline
 a^2 + 2ab + b^2 \\
 a + b \\
 \hline
 a^2 + 2a^2b + ab^2 \\
 \quad a^2b + 2ab^2 + b^3 \\
 \hline
 a^3 + 3a^2b + 3ab^2 + b^3
 \end{array}$$

$$\begin{array}{r}
 88. \quad 2x - 3y \\
 2x - 3y \\
 \hline
 4x^2 - 6xy \\
 \quad - 6xy + 9y^2 \\
 \hline
 4x^2 - 12xy + 9y^2 \\
 2x - 3y \\
 \hline
 8x^3 - 24x^2y + 18xy^2 \\
 \quad - 12x^2y + 36xy^2 - 27y^3 \\
 \hline
 8x^3 - 36x^2y + 54xy^2 - 27y^3
 \end{array}$$

89.

$$\begin{array}{r}
 x - y \\
 x - y \\
 \hline
 x^2 - xy \\
 \quad - xy + y^2 \\
 \hline
 x^2 - 2xy + y^2 \\
 x - y \\
 \hline
 x^3 - 2x^2y + xy^2 \\
 \quad - x^2y + 2xy^2 - y^3 \\
 \hline
 x^3 - 3x^2y + 3xy^2 - y^3 \\
 x - y \\
 \hline
 x^4 - 3x^3y + 3x^2y^2 - xy^3 \\
 \quad - x^3y + 3x^2y^2 - 3xy^3 + y^4 \\
 \hline
 x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4
 \end{array}$$

90.

$$\begin{array}{r}
 2a + 5b \\
 2a + 5b \\
 \hline
 4a^2 + 10ab \\
 \quad 10ab + 25b^2 \\
 \hline
 4a^2 + 20ab + 25b^2 \\
 2a + 5b \\
 \hline
 8a^3 + 40a^2b + 50ab^2 \\
 \quad 20a^2b + 100ab^2 + 125b^3 \\
 \hline
 8a^3 + 60a^2b + 150ab^2 + 125b^3 \\
 2a + 5b \\
 \hline
 16a^4 + 120a^3b + 800a^2b^2 + 250ab^3 \\
 \quad 40a^3b + 300a^2b^2 + 750ab^3 + 625b^4 \\
 \hline
 16a^4 + 160a^3b + 600a^2b^2 + 1000ab^3 + 625b^4
 \end{array}$$

$$\begin{array}{r}
 91. \quad x + 4y \\
 x + 4y \\
 \hline
 x^2 + 4xy \\
 \quad 4xy + 16y^2 \\
 \hline
 x^2 + 8xy + 16y^2
 \end{array}$$

$$\begin{array}{r}
 92. \quad 3x - 5 \\
 3x - 5 \\
 \hline
 9x^2 - 15x \\
 \quad - 15x + 25 \\
 \hline
 9x^2 - 30x + 25 \\
 3x - 5 \\
 \hline
 27x^3 - 90x^2 + 75x \\
 \quad - 45x^2 + 150x - 125 \\
 \hline
 27x^3 - 135x^2 + 225x - 125
 \end{array}$$

$$\begin{array}{r}
 93. \quad 2a - b \\
 2a - b \\
 \hline
 4a^2 - 2ab \\
 \quad - 2ab + b^2 \\
 \hline
 4a^2 - 4ab + b^2 \\
 2a - b \\
 \hline
 8a^3 - 8a^2b + 2ab^2 \\
 \quad - 4a^2b + 4ab^2 - b^3 \\
 \hline
 8a^3 - 12a^2b + 6ab^2 - b^3 \\
 2a - b \\
 \hline
 16a^4 - 24a^3b + 12a^2b^2 - 2ab^3 \\
 \quad - 8a^3b + 12a^2b^2 - 6ab^3 + b^4 \\
 \hline
 16a^4 - 32a^3b + 24a^2b^2 - 8ab^3 + b^4
 \end{array}$$

$$\begin{array}{r}
 94. \quad 2x - y \\
 2x - y \\
 \hline
 4x^2 - 2xy \\
 \quad - 2xy + y^2 \\
 \hline
 4x^2 - 4xy + y^2 \\
 x + y \\
 \hline
 4x^3 - 4x^2y + xy^2 \\
 \quad 4x^2y - 4xy^2 + y^3 \\
 \hline
 4x^3 - 3xy^2 + y^3
 \end{array}$$

$$\begin{array}{r}
 95. \quad a - 2b \\
 a - 2b \\
 \hline
 a^3 - 2ab \\
 \quad - 2ab + 4b^2 \\
 \hline
 a^3 - 4ab + 4b^2 \\
 a - 2b \\
 \hline
 a^3 - 4a^2b + 4ab^2 \\
 \quad - 2a^2b + 8ab^2 - 8b^3 \\
 \hline
 a^3 - 6a^2b + 12ab^2 - 8b^3 \\
 a + 2b \\
 \hline
 a^4 - 6a^3b + 12a^2b^2 - 8ab^3 \\
 \quad 2a^3b - 12a^2b^2 + 24ab^3 - 16b^4 \\
 \hline
 a^4 - 4a^3b + 16ab^3 - 16b^4
 \end{array}$$

$$\begin{array}{r}
 96. \quad x - 3y \\
 x - 3y \\
 \hline
 x^2 - 3xy \\
 \quad - 3xy + 9y^2 \\
 \hline
 x^2 - 6xy + 9y^2 \\
 x^2 + 6xy - 9y^2 \\
 \hline
 x^4 - 6x^2y + 9x^2y^2 \\
 \quad 6x^2y - 36x^2y^2 + 54xy^3 \\
 \quad - 9x^2y^2 + 54xy^3 - 81y^4 \\
 \hline
 x^4 - 36x^2y^2 + 108xy^3 - 81y^4
 \end{array}$$

EXERCISE V.

21. $x+2)x^3+12x+20(x+10$
 $\frac{x^3+2x}{10x+20}$
 $10x+20$
22. $y+4)y^2+14y+40(y+10$
 $\frac{y^3+4y}{10y+40}$
 $10y+40$
23. $x-12)x^2-x-182(x+11$
 $\frac{x^2-12x}{11x-182}$
 $11x-182$
24. $x+7)x^2-3x-70(x-10$
 $\frac{x^2+7x}{-10x-70}$
 $-10x-70$
25. $2a-6)2a^2-14a+24(a-4$
 $\frac{2a^2-6a}{-8a+24}$
 $-8a+24$
26. $3a-5)3a^2-26a+35(a-7$
 $\frac{3a^2-5a}{-21a+35}$
 $-21a+35$
27. $2x-5)2x^2+15x-50(x+10$
 $\frac{2x^2-5x}{20x-50}$
 $20x-50$
28. $3x+7)3x^2-5x-28(x-4$
 $\frac{3x^2+7x}{-12x-28}$
 $-12x-28$
29. $3x-2y)3x^2-11xy+6y^2(x-3y$
 $\frac{3x^2-2xy}{-9xy+6y^2}$
 $-9xy+6y^2$
30. $4a+b)12a^2-ab-b^2(3a-b$
 $\frac{12a^2+8ab}{-4ab-b^2}$
 $-4ab-b^2$
31. $x+7)x^3+3x^2-23x+35(x^2-4x+5$
 $\frac{x^3+7x^2}{-4x^2-23x}$
 $-4x^2-23x$
 $5x+35$
 $5x+35$
32. $x-5)x^3-12x^2+27x+40(x^2-7x-8$
 $\frac{x^3-5x^2}{-7x^2+27x}$
 $-7x^2+27x$
 $-7x^2+35x$
 $-8x+40$
 $-8x+40$
33. $2a+8)2a^3-7a^2-8a+18(a^2-5a+6$
 $\frac{2a^3+8a^2}{-10a^2-8a}$
 $-10a^2-8a$
 $-10a^2-16a$
 $12a+18$
 $12a+18$

34.
$$\begin{array}{r} 3a-4) 3a^3+17a^2-43a+20(a^2+7a-5) \\ \underline{3a^3-4a^2} \\ 21a^2-43a \\ \underline{21a^2-28a} \\ -15a+20 \\ \underline{-15a+20} \end{array}$$
35.
$$\begin{array}{r} 2y+5) 4y^3-10y^2-62y-30(2y^2-10y-6) \\ \underline{4y^3+10y^2} \\ -20y^2-62y \\ \underline{-20y^2-50y} \\ -12y-30 \\ \underline{-12y-30} \end{array}$$
36.
$$\begin{array}{r} 3x+4y) 3x^3-2x^2y-5xy^2+4y^3(x^2-2xy+y^2) \\ \underline{3x^3+4x^2y} \\ -6x^2y-5xy^2 \\ \underline{-6x^2y-8xy^2} \\ 3xy^2+4y^3 \\ \underline{+3xy^2+4y^3} \end{array}$$
37.
$$\begin{array}{r} 3x-4a) 3x^3-16ax^2+28a^2x-16a^3(x^2-4ax+4a^2) \\ \underline{3x^3-4ax^2} \\ -12ax^2+28a^2x \\ \underline{-12ax^2+16a^2x} \\ 12a^2x-16a^3 \\ \underline{12a^2x-16a^3} \end{array}$$
38.
$$\begin{array}{r} 2x+3y) 6x^3-x^2y-23xy^2-12y^3(3x^2-5xy-4y^2) \\ \underline{6x^3+9x^2y} \\ -10x^2y-23xy^2 \\ \underline{-10x^2y-15xy^2} \\ -8xy^2-12y^3 \\ \underline{-8xy^2-12y^3} \end{array}$$
39.
$$\begin{array}{r} 2x-7) 2x^4-17x^3+41x^2-29x+28(x^3-5x^2+3x-4) \\ \underline{2x^4-7x^3} \\ -10x^3+41x^2 \\ \underline{-10x^3+35x^2} \\ 6x^2-29x \\ \underline{6x^2-21x} \\ -8x+28 \\ \underline{-8x+28} \end{array}$$
40.
$$\begin{array}{r} 4x-5y) 8x^4+2x^3y-31x^2y^2+40xy^3-25y^4(2x^3+3x^2y-4xy^2+5y^3) \\ \underline{8x^4-10x^3y} \\ 12x^3y-31x^2y^2 \\ \underline{12x^3y-15x^2y^2} \\ -16x^2y^2+40xy^3 \\ \underline{-16x^2y^2+20xy^3} \\ 20xy^3-25y^4 \\ \underline{20xy^3-25y^4} \end{array}$$

41.
$$\begin{array}{r} 3x^2 + 4x + 5 \quad 6x^4 + 23a^3 + 42x^2 + 41x + 20(2x^2 + 5x + 4) \\ \underline{6x^4 + 8x^2 + 10x^2} \\ 15x^2 + 82x^2 + 41x \\ \underline{15x^2 + 20x^2 + 25x} \\ 12x^2 + 16x + 20 \\ \underline{12x^2 + 16x + 20} \end{array}$$
42.
$$\begin{array}{r} 2x^2 - 5x + 6 \quad 6x^4 - 23x^2 + 32x^2 - 9x - 18(8x^2 - 4x - 3) \\ \underline{6x^4 - 15x^2 + 18x^2} \\ - 8x^2 + 14x^2 - 9x \\ \underline{- 8x^2 + 20x^2 - 24x} \\ - 6x^2 + 15x - 18 \\ \underline{- 6x^2 + 15x - 18} \end{array}$$
43.
$$\begin{array}{r} 4x^2 - 5x + 10 \quad 20x^4 - 83x^2 + 72x^2 - 35x + 30(5x^2 - 2x + 3) \\ \underline{20x^4 - 25x^2 + 50x^2} \\ - 8x^2 + 22x^2 - 35x \\ \underline{- 8x^2 + 10x^2 - 20x} \\ 12x^2 - 15x + 30 \\ \underline{12x^2 - 15x + 30} \end{array}$$
44.
$$\begin{array}{r} y^2 - 5y - 12 \quad 2y^4 - 16y^2 + 2y^2 + 92y + 48(2y^2 - 6y - 4) \\ \underline{2y^4 - 10y^2 - 24y^2} \\ - 6y^2 + 26y^2 + 92y \\ \underline{- 6y^2 + 30y^2 + 72y} \\ - 4y^2 + 20y + 48 \\ \underline{- 4y^2 + 20y + 48} \end{array}$$
45.
$$\begin{array}{r} a^2 - 6a - 10 \quad 3a^4 - 23a^2 - 5a^2 + 80a + 50(3a^2 - 5a - 5) \\ \underline{3a^4 - 18a^2 - 30a^2} \\ - 5a^2 + 25a^2 + 80a \\ \underline{- 5a^2 + 30a^2 + 50a} \\ - 5a^2 + 30a + 50 \\ \underline{- 5a^2 + 30a + 50} \end{array}$$
46.
$$\begin{array}{r} x^2 - 5xy + 7y^2 \quad x^4 - 8x^2y + 21x^2y^2 - 16xy^3 - 7y^4(x^2 - 3xy - y^2) \\ \underline{x^4 - 5x^2y + 7x^2y^2} \\ - 3x^2y + 14x^2y^2 - 16xy^3 \\ \underline{- 3x^2y + 15x^2y^2 - 21xy^3} \\ - x^2y^2 + 5xy^3 - 7y^4 \\ \underline{- x^2y^2 + 5xy^3 - 7y^4} \end{array}$$
47.
$$\begin{array}{r} x^2 - 4ax - 3a^2 \quad x^4 - 9ax^2 + 12a^2x^2 + 35ax^2 + 15a^4(x^2 - 5ax - 5a^2) \\ \underline{x^4 - 4ax^2 - 3a^2x^2} \\ - 5ax^2 + 15a^2x^2 + 35ax^2 \\ \underline{- 5ax^2 + 20a^2x^2 + 15ax^2} \\ - 5a^2x^2 + 20ax^2 + 15a^4 \\ \underline{- 5a^2x^2 + 20ax^2 + 15a^4} \end{array}$$

$$48. \quad \begin{array}{r} 2a^3 - 4ab - 5b^3) 4a^4 - 16a^3b - 4a^2b^2 + 40ab^3 + 25b^4 (2a^2 - 4ab - 5b^2 \\ \underline{4a^4 - 8a^3b - 10a^2b^3} \end{array}$$

$$\begin{array}{r} - 8a^3b + 6a^2b^2 + 40ab^3 \\ - 8a^3b + 16a^2b^2 + 20ab^3 \\ \hline - 10a^2b^2 + 20ab^3 + 25b^4 \\ - 10a^2b^2 + 20ab^3 + 25b^4 \end{array}$$

$$49. \quad \begin{array}{r} 4y^2 - 7by + 8b^2) 4y^4 - 15by^3 + 26b^2y^2 - 23b^3y + 8b^4 (y^2 - 2by + b^2 \\ \underline{4y^4 - 7by^3 + 8b^2y^2} \end{array}$$

$$\begin{array}{r} - 8by^3 + 18b^2y^2 - 23b^3y \\ - 8by^3 + 14b^2y^2 - 16b^3y \\ \hline 4b^2y^2 - 7b^3y + 8b^4 \\ 4b^2y^2 - 7b^3y + 8b^4 \end{array}$$

$$50. \quad \begin{array}{r} 5x^2 - 4xy + 8y^2) 5x^4 - 14x^3y + 31x^2y^2 - 22xy^3 + 12y^4 (x^2 - 2xy + 4y^2 \\ \underline{5x^4 - 4x^3y + 3x^2y^2} \end{array}$$

$$\begin{array}{r} - 10x^3y + 28x^2y^2 - 22xy^3 \\ - 10x^3y + 8x^2y^2 - 6xy^3 \\ \hline 20x^2y^2 - 16xy^3 + 12y^4 \\ 20x^2y^2 - 16xy^3 + 12y^4 \end{array}$$

$$51. \quad \begin{array}{r} 3a^3 + 6ab - b^3) 6a^4 + 21a^3b + 31a^2b^2 + 27ab^3 - 5b^4 (2a^2 + 3ab + 5b^2 \\ \underline{6a^4 + 12a^3b - 2a^2b^3} \end{array}$$

$$\begin{array}{r} 9a^3b + 38a^2b^2 + 27ab^3 \\ 9a^3b + 18a^2b^2 - 3ab^3 \\ \hline 15a^2b^2 + 30ab^3 - 5b^4 \\ 15a^2b^2 + 30ab^3 - 5b^4 \end{array}$$

$$52. \quad \begin{array}{r} 2a^3 - 6ab + 5b^3) 2a^4 - 2a^3b - 5a^2b^2 + 4ab^3 + 5b^4 (a^2 + 2ab + b^2 \\ \underline{2a^4 - 6a^3b + 5a^2b^2} \end{array}$$

$$\begin{array}{r} 4a^3b - 10a^2b^2 + 4ab^3 \\ 4a^3b - 12a^2b^2 + 10ab^3 \\ \hline 2a^2b^2 - 6ab^3 + 5b^4 \\ 2a^2b^2 - 6ab^3 + 5b^4 \end{array}$$

$$53. \quad \begin{array}{r} 3x^2 + 4ax - 5a^2) 6x^4 - 7ax^3 - 36a^2x^2 + 17a^3x + 10a^4 (2x^2 - 5ax - 2a^2 \\ \underline{6x^4 + 8ax^3 - 10a^2x^2} \end{array}$$

$$\begin{array}{r} - 15ax^3 - 26a^2x^2 + 17a^3x \\ - 15ax^3 - 20a^2x^2 + 25a^3x \\ \hline - 6a^2x^2 - 8a^3x + 10a^4 \\ - 6a^2x^2 - 8a^3x + 10a^4 \end{array}$$

$$54. \quad \begin{array}{r} 9x^3 - 7xy + 2y^2) 27x^4 - 3x^3y - 35x^2y^2 + 25xy^3 - 6y^4 (3x^2 + 2xy - 3y^2 \\ \underline{27x^4 - 21x^3y + 6x^2y^2} \end{array}$$

$$\begin{array}{r} 18x^3y - 41x^2y^2 + 25xy^3 \\ 18x^3y - 14x^2y^2 + 4xy^3 \\ \hline - 27x^2y^2 + 21xy^3 - 6y^4 \\ - 27x^2y^2 + 21xy^3 - 6y^4 \end{array}$$

EXERCISE V.

17.

$$\begin{array}{r}
 55. \quad 4x^2 + 5xy - y^2) 8x^4 - 2x^3y - 21x^2y^2 - 2xy^3 + y^4 (2x^2 - 3xy - y^2 \\
 \underline{8x^4 + 10x^3y - 2x^2y^2} \\
 \quad -12x^3y - 19x^2y^2 - 2xy^3 \\
 \quad \underline{-12x^3y - 15x^2y^2 + 3xy^3} \\
 \qquad \quad -4x^2y^2 - 5xy^3 + y^4 \\
 \qquad \quad \underline{-4x^2y^2 - 5xy^3 + y^4}
 \end{array}$$

$$\begin{array}{r}
 56. \quad 8x^2 - 3xy + 2y^2) 16x^4 - 46x^3y - 21x^2y^2 + 5xy^3 - 10y^4 (2x^2 - 5xy - 5y^2 \\
 \underline{16x^4 - 6x^3y + 4x^2y^2} \\
 \quad -40x^3y - 25x^2y^2 + 5xy^3 \\
 \quad \underline{-40x^3y + 15x^2y^2 - 10xy^3} \\
 \qquad \quad -40x^2y^2 + 15xy^3 - 10y^4 \\
 \qquad \quad \underline{-40x^2y^2 + 15xy^3 - 10y^4}
 \end{array}$$

$$\begin{array}{r}
 57. \quad 7a^2 - 5ab + 6b^2) 14a^4 - 45a^3b + 16a^2b^2 - 15ab^3 - 18b^4 (2a^2 - 5ab - 3b^2 \\
 \underline{14a^4 - 10a^3b + 12a^2b^2} \\
 \quad -35a^3b + 4a^2b^2 - 15ab^3 \\
 \quad \underline{-35a^3b + 25a^2b^2 - 30ab^3} \\
 \qquad \quad -21a^2b^2 + 15ab^3 - 18b^4 \\
 \qquad \quad \underline{-21a^2b^2 + 15ab^3 - 18b^4}
 \end{array}$$

$$\begin{array}{r}
 58. \quad 3x^2 - 5ax - 3a^2) 9x^4 - 9ax^3 - 13a^2x^2 - 16a^3x - 6a^4 (3x^2 + 2ax + 2a^2 \\
 \underline{9x^4 - 15ax^3 - 9a^2x^2} \\
 \quad 6ax^3 - 4a^2x^2 - 16a^3x \\
 \quad \underline{6ax^3 - 10a^2x^2 - 6a^3x} \\
 \qquad \quad 6a^2x^2 - 10a^3x - 6a^4 \\
 \qquad \quad \underline{6a^2x^2 - 10a^3x - 6a^4}
 \end{array}$$

$$\begin{array}{r}
 59. \quad x^3 - 5x - 6) x^5 - 10x^4 + 17x^3 + 41x^2 + 7x - 6 (x^2 - 5x^2 - 2x + 1 \\
 \underline{x^5 - 5x^4 - 6x^3} \\
 \quad -5x^4 + 23x^3 + 41x^2 \\
 \quad \underline{-5x^4 + 25x^3 + 30x^2} \\
 \qquad \quad -2x^3 + 11x^2 + 7x \\
 \qquad \quad \underline{-2x^3 + 10x^2 + 12x} \\
 \qquad \qquad \quad x^2 - 5x - 6 \\
 \qquad \qquad \quad \underline{x^2 - 5x - 6}
 \end{array}$$

$$\begin{array}{r}
 60. \quad x^2 - 3xy + 2y^2) x^5 - 5x^4y + 11x^3y^2 - 14x^2y^3 + 9xy^4 - 2y^5 (x^3 - 2x^2y + 3xy^2 - y^3 \\
 \underline{x^5 - 3x^4y + 2x^3y^2} \\
 \quad -2x^4y + 9x^3y^2 - 14x^2y^3 \\
 \quad \underline{-2x^4y + 6x^3y^2 - 4x^2y^3} \\
 \qquad \quad 3x^3y^2 - 10x^2y^3 + 9xy^4 \\
 \qquad \quad \underline{3x^3y^2 - 9x^2y^3 + 6xy^4} \\
 \qquad \qquad \quad -x^2y^3 + 3xy^4 - 2y^5 \\
 \qquad \qquad \quad \underline{-x^2y^3 + 3xy^4 - 2y^5}
 \end{array}$$

$$\begin{array}{r}
 61. \quad a + b) a^3 - b^3 (a - b \\
 \underline{a^3 + ab^2} \\
 \quad -ab - b^3
 \end{array}$$

$$\begin{array}{r}
 62. \quad 2x - 3y) 4x^3 - 9y^2 (2x + 3y \\
 \underline{4x^2 - 6xy} \\
 \quad 6xy - 9y^2
 \end{array}$$

$$63. \quad \begin{array}{r} a+b \overline{) a^3+b^3} \quad (a^2-ab+b^2) \\ \underline{a^3+a^2b} \\ -a^2b+b^3 \\ \underline{-a^2b-ab^3} \\ ab^3+b^3 \\ \underline{ab^3+b^3} \\ 0 \end{array} \quad 64. \quad \begin{array}{r} a-b \overline{) a^3-b^3} \quad (a^2+ab+b^2) \\ \underline{a^3-a^2b} \\ a^2b-b^3 \\ \underline{a^2b-ab^3} \\ ab^3-b^3 \\ \underline{ab^3-b^3} \\ 0 \end{array}$$

$$65. \quad \begin{array}{r} 3x-2a \overline{) 27x^3-8a^3} \quad (9x^2+6ax+4a^2) \\ \underline{27x^3-18ax^2} \\ 18ax^2-8a^3 \\ \underline{18ax^2-12a^2x} \\ 12a^2x-8a^3 \\ \underline{12a^2x-8a^3} \\ 0 \end{array}$$

$$66. \quad \begin{array}{r} 2a-3b \overline{) 16a^4-81b^4} \quad (8a^3+12a^2b+18ab^2+27b^3) \\ \underline{16a^4-24a^3b} \\ 24a^3b-81b^4 \\ \underline{24a^3b-36a^2b^2} \\ 36a^2b^2-81b^4 \\ \underline{36a^2b^2-54ab^3} \\ 54ab^3-81b^4 \\ \underline{54ab^3-81b^4} \\ 0 \end{array}$$

$$67. \quad \begin{array}{r} 2a+3b \overline{) 16a^4-81b^4} \quad (8a^3-12a^2b+18ab^2-27b^3) \\ \underline{16a^4+24a^3b} \\ -24a^3b-81b^4 \\ \underline{-24a^3b-36a^2b^2} \\ 36a^2b^2-81b^4 \\ \underline{36a^2b^2+54ab^3} \\ -54ab^3-81b^4 \\ \underline{-54ab^3-81b^4} \\ 0 \end{array}$$

$$68. \quad \begin{array}{r} x^3-2y^2 \overline{) x^6-8y^6} \quad (x^3+2x^2y^2+4y^4) \\ \underline{x^6-2x^4y^2} \\ 2x^4y^2-8y^6 \\ \underline{2x^4y^2-4x^2y^4} \\ 4x^2y^4-8y^6 \\ \underline{4x^2y^4-8y^6} \\ 0 \end{array}$$

$$69. \quad \begin{array}{r} x^2-4x+16 \overline{) x^3+64} \quad (x+4) \\ \underline{x^3-4x^2+16x} \\ 4x^2-16x+64 \\ \underline{4x^2-16x+64} \\ 0 \end{array} \quad 70. \quad \begin{array}{r} y^3+3y+9 \overline{) y^3-27} \quad (y-3) \\ \underline{y^3+3y^2+9y} \\ -3y^2-9y-27 \\ \underline{-3y^2-9y-27} \\ 0 \end{array}$$

$$71. \quad \begin{array}{r} x^3-xy+y^3 \overline{) x^4+x^2y^3+y^4} \quad (x^2+xy+y^2) \\ \underline{x^4-x^2y+x^2y^2} \\ x^2y+y^4 \\ \underline{x^2y-x^2y^2+xy^3} \\ x^2y^3-xy^3+y^4 \\ \underline{x^2y^3-xy^3+y^4} \\ 0 \end{array}$$

EXERCISE V.

19

72.
$$\begin{array}{r} a^2 + 4a + 16 \quad a^4 + 16a^2 + 256 \quad (a^3 - 4a + 16 \\ \underline{a^4 + 4a^3 + 16a^2} \\ - 4a^3 + 256 \\ \underline{- 4a^3 - 16a^2 - 64a} \\ 16a^2 + 64a + 256 \\ \underline{16a^2 + 64a + 256} \end{array}$$
73.
$$\begin{array}{r} a^4 - a^2b^2 + b^4 \quad a^5 + a^4b^4 + b^8 \quad (a^4 + a^2b^2 + b^4 \\ \underline{a^5 - a^3b^2 + a^2b^4} \\ a^6b^2 + b^8 \\ \underline{a^6b^2 - a^4b^4 + a^2b^6} \\ a^4b^4 - a^2b^6 + b^8 \\ \underline{a^4b^4 - a^2b^6 + b^8} \end{array}$$
74.
$$\begin{array}{r} x^2 + xy + y^2 \quad x^5 - x^3y^2 - x^2y^3 + y^5 \quad (x^3 - x^2y - xy^2 + y^3 \\ \underline{x^5 + x^4y + x^3y^2} \\ - x^4y - 2x^3y^2 - x^2y^3 + y^5 \\ \underline{- x^4y - x^3y^2 - x^2y^3} \\ - x^3y^2 + y^5 \\ \underline{- x^3y^2 - x^2y^3 - xy^4} \\ x^2y^3 + xy^4 + y^5 \\ \underline{x^2y^3 + xy^4 + y^5} \end{array}$$
75.
$$\begin{array}{r} a^2 - 4b^2 \quad a^5 - 4a^3b^2 - 8a^2b^3 + 32b^5 \quad (a^3 - 8b^3 \\ \underline{a^5 - 4a^3b^2} \\ - 8a^2b^3 + 32b^5 \\ \underline{- 8a^2b^3 + 32b^5} \end{array}$$
76.
$$\begin{array}{r} x^2 - xy + y^2 \quad x^6 - y^6 \quad (x^4 + x^2y - xy^2 - y^4 \\ \underline{x^6 - x^3y + x^2y^2} \\ x^3y - x^4y^2 - y^6 \\ \underline{x^3y - x^4y^2 + x^2y^3} \\ - x^3y^3 - y^6 \\ \underline{- x^3y^3 + x^2y^4 - xy^5} \\ - x^2y^4 + xy^5 - y^6 \\ \underline{- x^2y^4 + xy^5 - y^6} \end{array}$$
77.
$$\begin{array}{r} a^3 + 2a + 4 \quad a^6 - 64 \quad (a^4 - 2a^3 + 8a - 16 \\ \underline{a^6 + 2a^5 + 4a^4} \\ - 2a^5 - 4a^4 - 64 \\ \underline{- 2a^5 - 4a^4 - 8a^3} \\ 8a^3 - 64 \\ \underline{8a^3 + 16a^2 + 32a} \\ - 16a^2 - 32a - 64 \\ \underline{- 16a^2 - 32a - 64} \end{array}$$
78.
$$\begin{array}{r} x^2 - 5x + 7 \quad x^4 - 8x^3 + 21x^2 - 16x - 7 \quad (x^2 - 3x - 1 \\ \underline{x^4 - 5x^3 + 7x^2} \\ - 3x^3 + 14x^2 - 16x \\ \underline{- 3x^3 + 15x^2 - 21x} \\ - x^2 + 5x - 7 \\ \underline{- x^2 + 5x - 7} \end{array}$$

79.

$$\begin{array}{r}
 5x^2 - 4x + 8 \quad 5x^4 - 14x^3 + 31x^2 - 22x + 12 \quad (x^2 - 2x + 4 \\
 \underline{5x^4 - 4x^3 + 8x^2} \\
 -10x^3 + 28x^2 - 22x \\
 \underline{10x^3 + 8x^2 - 6x} \\
 20x^2 - 16x + 12 \\
 \underline{20x^2 - 16x + 12} \\
 0
 \end{array}$$

80.

$$\begin{array}{r}
 x^2 + 2x + 1 \quad 2x^4 - 2x^3 - 5x^2 + 4x + 5 \quad (2x^2 - 6x + 5 \\
 \underline{2x^4 + 4x^3 + 2x^2} \\
 -6x^3 - 7x^2 + 4x \\
 \underline{-6x^3 - 12x^2 - 6x} \\
 5x^2 + 10x + 5 \\
 \underline{5x^2 + 10x + 5} \\
 0
 \end{array}$$

81.

$$\begin{array}{r}
 3x^2 + 2x + 2 \quad 9x^4 - 9x^3 - 13x^2 - 16x - 6 \quad (3x^2 - 5x - 3 \\
 \underline{9x^4 + 6x^3 + 6x^2} \\
 -15x^3 - 19x^2 - 16x \\
 \underline{-15x^3 - 10x^2 - 10x} \\
 -9x^2 - 6x - 6 \\
 \underline{-9x^2 - 6x - 6} \\
 0
 \end{array}$$

82.

$$\begin{array}{r}
 x^3 + xy + y^2 \quad x^5 + x^4y + y^5 \quad (x^5 - x^4y + x^3y^3 - xy^5 + y^6 \\
 \underline{x^5 + x^4y + x^3y^2} \\
 -x^4y - x^3y^2 + x^4y^4 + y^5 \\
 \underline{-x^4y - x^3y^2 - x^3y^3} \\
 x^4y^3 + x^4y^4 + y^5 \\
 \underline{x^4y^3 + x^4y^4 + x^3y^5} \\
 -x^3y^3 + y^5 \\
 \underline{-x^3y^3 - x^2y^5 - xy^7} \\
 x^2y^5 + xy^7 + y^8 \\
 \underline{x^2y^5 + xy^7 + y^8} \\
 0
 \end{array}$$

83.

$$\begin{array}{r}
 x^3 + 2x^2y + 2xy^2 + y^3 \quad x^6 - y^6 \quad (x^3 - 2x^2y + 2xy^2 - y^3 \\
 \underline{x^6 + 2x^5y + 2x^4y^2 + x^3y^3} \\
 -2x^5y - 2x^4y^2 - x^3y^3 - y^6 \\
 \underline{-2x^5y - 4x^4y^2 - 4x^3y^3 - 2x^2y^4} \\
 2x^4y^3 + 3x^3y^3 + 2x^2y^4 - y^6 \\
 \underline{2x^4y^3 + 4x^3y^3 + 4x^2y^4 + 2xy^5} \\
 -x^3y^3 - 2x^2y^4 - 2xy^5 - y^6 \\
 \underline{-x^3y^3 - 2x^2y^4 - 2xy^5 - y^6} \\
 0
 \end{array}$$

84.

$$\begin{array}{r}
 x^3 - xy + y^3 \quad x^6 - x^4y^2 + 3x^3y^3 - x^2y^4 + y^6 \quad (x^3 + x^2y - x^2y^2 + xy^3 + y^4 \\
 \underline{x^6 - x^5y + x^4y^2} \\
 x^5y - 2x^4y^2 + 3x^3y^3 - x^2y^4 + y^6 \\
 \underline{x^5y - x^4y^2 + x^3y^3} \\
 -x^4y^2 + 2x^3y^3 - x^2y^4 + y^6 \\
 \underline{-x^4y^2 + x^3y^3 - x^2y^4} \\
 +x^3y^3 + y^6 \\
 \underline{x^3y^3 - x^2y^4 + xy^5} \\
 x^2y^4 - xy^5 + y^6 \\
 \underline{x^2y^4 - xy^5 + y^6} \\
 0
 \end{array}$$

$$\begin{array}{r}
 85. \quad a^3 + 2a^2y + 2ay^2 + y^3) a^5 - a^2y^2 - a^2y^3 + y^5 \quad (a^2 - 2ay + y^2 \\
 \underline{a^5 + 2a^4y + 2a^3y^2 + a^2y^3} \\
 -2a^4y - 3a^3y^2 - 2a^2y^3 + y^5 \\
 \underline{-2a^4y - 4a^3y^2 - 4a^2y^3 - 2ay^4} \\
 a^3y^2 + 2a^2y^3 + 2ay^4 + y^5 \\
 \underline{a^3y^2 + 2a^2y^3 + 2ay^4 + y^5} \\
 0
 \end{array}$$

$$\begin{array}{r}
 86. \quad a + b + c) a^2 + 2ab + 2ac + b^2 + 2bc + c^2 (a + b + c \\
 \underline{a^2 + ab + ac} \\
 ab + ac + b^2 + 2bc + c^2 \\
 \underline{ab + b^2 + bc} \\
 ac + bc + c^2 \\
 \underline{ac + bc + c^2} \\
 0
 \end{array}$$

$$\begin{array}{r}
 87. \quad a + b + c) a^3 - 3abc + b^2 + c^3 (a^2 - ab - ac + b^2 - bc + c^2 \\
 \underline{a^3 + a^2b + a^2c} \\
 -a^2b - a^2c - 3abc + b^2 + c^3 \\
 \underline{-a^2b - ab^2 - abc} \\
 -a^2c + ab^2 - 2abc + b^2 + c^3 \\
 \underline{-a^2c - abc - ac^2} \\
 ab^2 - abc + ac^2 + b^2 + c^3 \\
 \underline{ab^2 + b^3 + b^2c} \\
 -abc + ac^2 - b^2c + c^3 \\
 \underline{-abc - b^2c - bc^2} \\
 ac^2 + bc^2 + c^3 \\
 \underline{ac^2 + bc^2 + c^3} \\
 0
 \end{array}$$

$$\begin{array}{r}
 88. \quad a - 2b + c) a^3 - 8b^3 + c^3 + 6abc (a^2 + 2ab - ac + 4b^2 + 2bc + c^2 \\
 \underline{a^3 - 2a^2b + a^2c} \\
 2a^2b - a^2c + 6abc - 8b^3 + c^3 \\
 \underline{2a^2b - 4ab^2 + 2abc} \\
 -a^2c + 4ab^2 + 4abc - 8b^3 + c^3 \\
 \underline{-a^2c + 2abc - ac^2} \\
 4ab^2 + 2abc + ac^2 - 8b^3 + c^3 \\
 \underline{4ab^2 - 8b^3 + 4b^2c} \\
 2abc + ac^2 - 4b^2c + c^3 \\
 \underline{2abc - 4b^2c + 2bc^2} \\
 ac^2 - 2bc^2 + c^3 \\
 \underline{ac^2 - 2bc^2 + c^3} \\
 0
 \end{array}$$

$$\begin{array}{r}
 89. \quad a + b - c) a^3 - b^3 + 2bc - c^3 (a - b + c \\
 \underline{a^3 + ab - ac} \\
 -ab + ac - b^2 + 2bc - c^3 \\
 \underline{-ab - b^2 + bc} \\
 ac + bc - c^3 \\
 \underline{ac + bc - c^3} \\
 0
 \end{array}$$

EXERCISE VI.

$$\begin{array}{r} 19. \qquad \qquad \qquad 100a^8 - 60a^4 + 9(10a^4 - 3) \\ \qquad \qquad \qquad 100a^8 \\ 20a^4 - 3 \overline{) \qquad \qquad \qquad} \begin{array}{r} - 60a^4 + 9 \\ - 60a^4 + 9 \end{array} \end{array}$$

EXERCISE VI.

23

20.

$$\begin{array}{r} 144a^3b^4 - 216ab^2 + 81(12ab^3 - 9 \\ 144a^3b^4 \\ 24ab^3 - 9 \overline{) \begin{array}{r} -216ab^3 + 81 \\ -216ab^3 + 81 \end{array}} \end{array}$$

21.

$$\begin{array}{r} 49x^4z^3 - 42a^3x^2z + 9a^4(7x^2z - 3a^2 \\ 49x^4z^3 \\ 14x^2z - 3a^2 \overline{) \begin{array}{r} -42a^3x^2z + 9a^4 \\ -42a^3x^2z + 9a^4 \end{array}} \end{array}$$

22.

$$\begin{array}{r} 25a^3b^4 - 20ab^3c^3 + 4c^6(5ab^3 - 2c^3 \\ 25a^3b^4 \\ 10ab^3 - 2c^3 \overline{) \begin{array}{r} -20ab^3c^3 + 4c^6 \\ -20ab^3c^3 + 4c^6 \end{array}} \end{array}$$

23.

$$\begin{array}{r} 49x^4 - 154x^2yz^3 + 121y^2z^4(7x^2 - 11yz^2 \\ 49x^4 \\ 14x^2 - 11yz^2 \overline{) \begin{array}{r} -154x^2yz^3 + 121y^2z^4 \\ -154x^2yz^3 + 121y^2z^4 \end{array}} \end{array}$$

24.

$$\begin{array}{r} 4 - 28x + 49x^3(2 - 7x \\ 4 \\ 4 - 7x \overline{) \begin{array}{r} -28x + 49x^3 \\ -28x + 49x^3 \end{array}} \end{array}$$

25.

$$\begin{array}{r} 1 - 6ax^3 + 9a^2x^4(1 - 3ax^2 \\ 1 \\ 2 - 3ax^2 \overline{) \begin{array}{r} -6ax^3 + 9a^2x^4 \\ -6ax^3 + 9a^2x^4 \end{array}} \end{array}$$

26.

$$\begin{array}{r} 16 - 24x + 9x^3(4 - 3x \\ 16 \\ 8 - 3x \overline{) \begin{array}{r} -24x + 9x^3 \\ -24x + 9x^3 \end{array}} \end{array} \quad \begin{array}{r} 9 - 30x^2 + 25x^4(3 - 5x^2 \\ 9 \\ 6 - 5x^2 \overline{) \begin{array}{r} -30x^2 + 25x^4 \\ -30x^2 + 25x^4 \end{array}} \end{array}$$

$$\begin{array}{r} 4 - 3x \\ 3 - 5x^2 \end{array}$$

27.

$$\begin{array}{r} 1 - 18x^3 + 81x^4(1 - 9x^2 \\ 1 \\ 2 - 9x^2 \overline{) \begin{array}{r} -18x^3 + 81x^4 \\ -18x^3 + 81x^4 \end{array}} \end{array} \quad \begin{array}{r} 9 - 30xy^2 + 25x^2y^4(3 - 5xy^2 \\ 9 \\ 6 - 5xy^2 \overline{) \begin{array}{r} -30xy^2 + 25x^2y^4 \\ -30xy^2 + 25x^2y^4 \end{array}} \end{array}$$

$$\begin{array}{r} 1 - 9x^2 \\ 3 - 5xy^2 \end{array}$$

24.

EXERCISE VI.

28.

$$\begin{array}{r}
 25a^2 - 30axyz + 9x^2y^2z^2 \quad (5a - 3xyz) \\
 25a^2 \\
 \hline
 10a - 3xyz \quad \begin{array}{l} - 30axyz + 9x^2y^2z^2 \\ - 30axyz + 9x^2y^2z^2 \end{array} \\
 \hline
 49c^2 - 28cz^2 + 4z^4 \quad (7c - 2z^2) \\
 49c^2 \\
 \hline
 14c - 2z^2 \quad \begin{array}{l} - 28cz^2 + 4z^4 \\ - 28cz^2 + 4z^4 \end{array} \quad \frac{5a - 3xyz}{7c - 2z^2}
 \end{array}$$

29.

$$\begin{array}{r}
 x^2 - 10x^4y^2 + 25x^{10}y^4 \quad (x - 5x^5y^2) \\
 x^2 \\
 \hline
 2x - 5x^6y^2 \quad \begin{array}{l} - 10x^4y^2 + 25x^{10}y^4 \\ - 10x^4y^2 + 25x^{10}y^4 \end{array} \\
 \hline
 16x^2 - 56x^4y^2 + 49x^6y^2 \quad (4x - 7x^3y^2) \\
 16x^2 \\
 \hline
 8x - 7x^3y^2 \quad \begin{array}{l} - 56x^4y^2 + 49x^6y^2 \\ - 56x^4y^2 + 49x^6y^2 \end{array} \quad \frac{x - 5x^5y^2}{4x - 7x^3y^2}
 \end{array}$$

30.

$$\begin{array}{r}
 9a^6x^{10} + 42a^2x^6 + 49x^2 \quad (3a^2x^5 + 7x) \\
 9a^6x^{10} \\
 \hline
 6a^2x^6 + 7x \quad \begin{array}{l} 42a^2x^6 + 49x^2 \\ 42a^2x^6 + 49x^2 \end{array} \\
 \hline
 25a^2x^2 - 80ax^4 + 64x^6 \quad (5ax - 8x^3) \\
 25a^2x^2 \\
 \hline
 10ax - 8x^3 \quad \begin{array}{l} - 80ax^4 + 64x^6 \\ - 80ax^4 + 64x^6 \end{array} \quad \frac{3a^2x^5 + 7x}{5ax - 8x^3}
 \end{array}$$

31.

$$\begin{array}{r}
 1 + 4x + 8x^2 + 8x^3 + 4x^4 \quad (1 + 2x + 2x^2) \\
 1 \\
 \hline
 2 + 2x \quad \begin{array}{l} 4x + 8x^2 \\ 4x + 8x^2 \end{array} \\
 \hline
 2 + 4x + 2x^2 \quad \begin{array}{l} 4x^2 + 8x^3 + 4x^4 \\ 4x^2 + 8x^3 + 4x^4 \end{array}
 \end{array}$$

32.

$$\begin{array}{r}
 x^4 - 4x^3 + 2x^2 + 4x + 1 \quad (x^2 - 2x - 1) \\
 x^4 \\
 \hline
 2x^2 - 2x \quad \begin{array}{l} - 4x^3 + 2x^2 \\ - 4x^3 + 2x^2 \end{array} \\
 \hline
 2x^2 - 4x - 1 \quad \begin{array}{l} - 2x^2 + 4x + 1 \\ - 2x^2 + 4x + 1 \end{array}
 \end{array}$$

33.

$$\begin{array}{r}
 x^4 + 2x^3 + 3x^2 + 2x + 1 \overline{) x^4 + x^3 + x^2 + x + 1} \\
 \underline{2x^3 + x^2} \\
 2x^3 + 2x + 1 \\
 \underline{2x^3 + 2x + 1} \\
 0
 \end{array}$$

34.

$$\begin{array}{r}
 16x^4 + 16x^3 + 12x^2 + 4x + 1 \overline{) 16x^4 + 16x^3 + 12x^2 + 4x + 1} \\
 \underline{16x^4} \\
 8x^3 + 2x^2 \\
 \underline{8x^3 + 4x^2} \\
 8x^2 + 4x + 1 \\
 \underline{8x^2 + 4x + 1} \\
 0
 \end{array}$$

35.

$$\begin{array}{r}
 x^4 - 6x^3 + 27x^2 - 54x + 81 \overline{) x^4 - 6x^3 + 27x^2 - 54x + 81} \\
 \underline{x^4 - 6x^3 + 27x^2} \\
 2x^3 - 6x + 9 \\
 \underline{2x^3 - 6x + 9} \\
 0
 \end{array}$$

36.

$$\begin{array}{r}
 25a^4 + 30a^3 - a^2 - 6a + 1 \overline{) 25a^4 + 30a^3 - a^2 - 6a + 1} \\
 \underline{25a^4 + 30a^3} \\
 10a^2 + 6a - 1 \\
 \underline{10a^2 + 6a - 1} \\
 0
 \end{array}$$

37.

$$\begin{array}{r}
 9a^5 - 42a^4 + 61a^3 - 28a^2 + 4 \overline{) 9a^5 - 42a^4 + 61a^3 - 28a^2 + 4} \\
 \underline{9a^5 - 42a^4 + 61a^3} \\
 6a^4 - 7a^3 \\
 \underline{6a^4 - 42a^3 + 61a^2} \\
 6a^4 - 14a^3 + 2 \\
 \underline{6a^4 - 42a^3 + 61a^2} \\
 12a^4 - 28a^3 + 4 \\
 \underline{12a^4 - 42a^3 + 61a^2} \\
 0
 \end{array}$$

38.

$$\begin{array}{r}
 121 + 66xy - 101x^2y^2 - 30x^3y^3 + 25x^4y^4 \overline{) 121 + 66xy - 101x^2y^2 - 30x^3y^3 + 25x^4y^4} \\
 \underline{121} \\
 22 + 3xy \\
 \underline{22 + 3xy} \\
 22 + 6xy - 5x^2y^2 \\
 \underline{22 + 6xy - 5x^2y^2} \\
 0
 \end{array}$$

39.
$$\begin{array}{r} 16 - 24ab^2 + 25a^2b^4 - 12a^3b^6 + 4a^4b^8 (4 - 3ab^2 + 2a^2b^4) \\ 16 \\ 8 - 3ab^2 \overline{) -24ab^2 + 25a^2b^4} \\ \underline{-24ab^2 + 9a^2b^4} \\ 8 - 6ab^2 + 2a^2b^4 \overline{) 16a^2b^4 - 12a^3b^6 + 4a^4b^8} \\ \underline{16a^2b^4 - 12a^3b^6 + 4a^4b^8} \end{array}$$
40.
$$\begin{array}{r} 81a^5 + 18a^6b - 71a^4b^2 - 8a^3b^3 + 16b^4 (9a^4 + a^2b^2 - 4b^2) \\ 81a^5 \\ 18a^4 + a^2b \overline{) 18a^6b - 71a^4b^2} \\ \underline{18a^6b + a^4b^2} \\ 18a^4 + 2a^2b - 4b^2 \overline{) -72a^4b^2 - 8a^2b^3 + 16b^4} \\ \underline{-72a^4b^2 - 8a^2b^3 + 16b^4} \end{array}$$
41.
$$\begin{array}{r} 4x^4 + 24x^3 - 108x + 81 (2x^2 + 6x - 9) \\ 4x^4 \\ 4x^3 + 6x \overline{) 24x^3} \\ \underline{24x^3 + 36x^2} \\ 4x^3 + 12x - 9 \overline{) -36x^2 - 108x + 81} \\ \underline{-36x^2 - 108x + 81} \end{array}$$
42.
$$\begin{array}{r} 64 - 192x^2 + 216x^3 + 81x^5 (8 - 12x^2 - 9x^4) \\ 64 \\ 16 - 12x^2 \overline{) -192x^2} \\ \underline{-192x^2 + 144x^4} \\ 16 - 24x^2 - 9x^4 \overline{) -144x^4 + 216x^6 + 81x^8} \\ \underline{-144x^4 + 216x^6 + 81x^8} \end{array}$$
43.
$$\begin{array}{r} x^4 - 14x^3 + 31x^2 + 126x + 81 (x^2 - 7x - 9) \\ x^4 \\ 2x^2 - 7x \overline{) -14x^3 + 31x^2} \\ \underline{-14x^3 + 49x^2} \\ 2x^2 - 14x - 9 \overline{) -18x^2 + 126x + 81} \\ \underline{-18x^2 + 126x + 81} \end{array}$$
44.
$$\begin{array}{r} x^4 - 6x^2 + x^3 + 24x + 16 (x^2 - 3x - 4) \\ x^4 \\ 2x^2 - 3x \overline{) -6x^2 + x^3} \\ \underline{-6x^2 + 9x^2} \\ 2x^2 - 6x - 4 \overline{) -8x^2 + 24x + 16} \\ \underline{-8x^2 + 24x + 16} \end{array}$$
45.
$$\begin{array}{r} x^5 - 4x^5 + 4x^4 - 18x^3 + 36x^2 + 81 (x^2 - 2x^2 - 9) \\ x^5 \\ 2x^3 - 2x^2 \overline{) -4x^5 + 4x^4} \\ \underline{-4x^5 + 4x^4} \\ 2x^3 - 4x^2 - 9 \overline{) -18x^3 + 36x^2 + 81} \\ \underline{-18x^3 + 36x^2 + 81} \end{array}$$

46.
$$\begin{array}{r} x^{12} - 18x^9 + 83x^6 - 18x^3 + 1 \quad (x^6 - 9x^3 + 1) \\ x^{12} \\ \hline 2x^6 - 9x^3 \quad \begin{array}{r} -18x^9 + 83x^6 \\ -18x^9 + 81x^6 \\ \hline 2x^6 - 18x^3 + 1 \end{array} \\ 2x^6 - 18x^3 + 1 \quad \begin{array}{r} 2x^6 - 18x^3 + 1 \\ 2x^6 - 18x^3 + 1 \\ \hline \end{array} \end{array}$$
47.
$$\begin{array}{r} 16 - 24b + 9b^2 + 8c - 6bc + c^2 \quad (4 - 3b + c) \\ 16 \\ \hline 8 - 3b \quad \begin{array}{r} -24b + 9b^2 \\ -24b + 9b^2 \\ \hline 8 - 6b + c \end{array} \\ 8 - 6b + c \quad \begin{array}{r} 8c - 6bc + c^2 \\ 8c - 6bc + c^2 \\ \hline \end{array} \end{array}$$
48.
$$\begin{array}{r} 64 - 64x + 16x^2 + 64y^2 - 32xy^2 + 16y^4 \quad (8 - 4x + 4y^2) \\ 64 \\ \hline 16 - 4x \quad \begin{array}{r} -64x + 16x^2 \\ -64x + 16x^2 \\ \hline 16 - 8x + 4y^2 \end{array} \\ 16 - 8x + 4y^2 \quad \begin{array}{r} 64y^2 - 32xy^2 + 16y^4 \\ 64y^2 - 32xy^2 + 16y^4 \\ \hline \end{array} \end{array}$$
49.
$$\begin{array}{r} 1 + 2x + 3x^2 + 4x^3 + 3x^4 + 2x^5 + x^6 \quad (1 + x + x^2 + x^3) \\ 1 \\ \hline 2 + x \quad \begin{array}{r} 2x + 3x^2 \\ 2x + x^2 \\ \hline 2 + 2x + x^2 \end{array} \\ 2 + 2x + x^2 \quad \begin{array}{r} 2x^2 + 4x^3 + 3x^4 \\ 2x^2 + 2x^3 + x^4 \\ \hline 2 + 2x + 2x^3 + x^5 \end{array} \\ 2 + 2x + 2x^3 + x^5 \quad \begin{array}{r} 2x^3 + 2x^4 + 2x^5 + x^6 \\ 2x^3 + 2x^4 + 2x^5 + x^6 \\ \hline \end{array} \end{array}$$
50.
$$\begin{array}{r} a^{12} + 4a^{10}x + 2a^8x^2 - 2a^6x^3 + 5a^4x^4 - 2a^2x^5 + x^6 \quad (a^6 + 2a^4x - a^2x^2 + x^3) \\ a^{12} \\ \hline 2a^6 + 2a^4x \quad \begin{array}{r} 4a^{10}x + 2a^8x^2 \\ 4a^{10}x + 4a^8x^2 \\ \hline 2a^6 + 4a^4x - a^2x^2 \end{array} \\ 2a^6 + 4a^4x - a^2x^2 \quad \begin{array}{r} -2a^6x^2 - 2a^4x^3 + 5a^4x^4 \\ -2a^6x^2 - 4a^6x^3 + a^4x^4 \\ \hline 2a^6x^3 + 4a^4x^4 - 2a^2x^5 + x^6 \end{array} \\ 2a^6 + 4a^4x - 2a^2x^3 + x^6 \quad \begin{array}{r} 2a^6x^3 + 4a^4x^4 - 2a^2x^5 + x^6 \\ 2a^6x^3 + 4a^4x^4 - 2a^2x^5 + x^6 \\ \hline \end{array} \end{array}$$
51.
$$\begin{array}{r} x^6 + 4x^5y + 2x^4y^2 + 9x^3y^4 - 4xy^5 + 4y^6 \quad (x^3 + 2x^2y - xy^2 + 2y^3) \\ x^6 \\ \hline 2x^3 + 2x^2y \quad \begin{array}{r} 4x^5y + 2x^4y^2 \\ 4x^5y + 4x^4y^2 \\ \hline 2x^3 + 4x^2y - xy^2 \end{array} \\ 2x^3 + 4x^2y - xy^2 \quad \begin{array}{r} -2x^4y^2 + 9x^3y^4 \\ -2x^4y^2 - 4x^3y^3 + x^2y^4 \\ \hline 2x^3 + 4x^2y - 2xy^2 + 2y^3 \end{array} \\ 2x^3 + 4x^2y - 2xy^2 + 2y^3 \quad \begin{array}{r} 4x^3y^3 + 8x^2y^4 - 4xy^5 + 4y^6 \\ 4x^3y^3 + 8x^2y^4 - 4xy^5 + 4y^6 \\ \hline \end{array} \end{array}$$

$$52. \quad \frac{64a^{12} - 128a^{10}b + 160a^8b^2 - 160a^6b^3 + 100a^4b^4 - 48a^2b^5 + 16b^6}{64a^{12}} (8a^6 - 8a^4b + 6a^2b^2 - 4b^3)$$

$$\begin{array}{l} 16a^6 - 8a^4b \\ \hline 16a^6 - 16a^4b + 6a^2b^2 \\ \hline 16a^6 - 16a^4b + 12a^2b^2 - 4b^3 \\ \hline \frac{96a^6b^2 - 160a^4b^3 + 100a^2b^4}{96a^6b^2 - 96a^4b^3 + 96a^2b^4} \\ \hline \frac{64a^6b^3 + 64a^4b^4 - 48a^2b^5 + 16b^6}{64a^6b^3 + 64a^4b^4 - 48a^2b^5 + 16b^6} \end{array}$$

$$53. \quad \frac{4 - 8xy^2 + 36x^2y^4 - 64x^3y^6 + 96x^4y^8 - 128x^5y^{10} + 64x^6y^{12}}{4} (2 - 2xy^2 + 8x^2y^4 - 8x^3y^6)$$

$$\begin{array}{l} 4 - 2xy^2 \\ \hline 4 - 4xy^2 + 8x^2y^4 \\ \hline 4 - 4xy^2 + 16x^2y^4 - 8x^3y^6 \\ \hline \frac{-8xy^2 + 36x^2y^4}{-8xy^2 + 4x^2y^4} \\ \hline \frac{32x^2y^4 - 64x^3y^6 + 96x^4y^8}{32x^2y^4 - 32x^3y^6 + 64x^4y^8} \\ \hline \frac{-32x^3y^6 + 32x^4y^8 - 128x^5y^{10} + 64x^6y^{12}}{-32x^3y^6 + 32x^4y^8 - 128x^5y^{10} + 64x^6y^{12}} \end{array}$$

$$54. \quad \frac{x^{16} - 2x^{14}y + 3x^{12}y^2 - 4x^{10}y^3 + 5x^8y^4 - 4x^6y^5 + 3x^4y^6 - 2x^2y^7 + y^8}{x^{16}} (x^2 - x^2y + x^2y^2 - x^2y^3 + y^4)$$

$$\begin{array}{l} 2x^8 - x^6y \\ \hline 2x^8 - 2x^6y + x^4y^2 \\ \hline 2x^8 - 2x^6y + 2x^4y^2 - x^2y^3 \\ \hline 2x^8 - 2x^6y + 2x^4y^2 - 2x^2y^3 + y^4 \\ \hline \frac{-2x^{14}y + 3x^{12}y^2}{-2x^{14}y + x^{12}y^2} \\ \hline \frac{2x^{12}y^2 - 4x^{10}y^3 + 5x^8y^4}{2x^{12}y^2 - 2x^{10}y^3 + x^8y^4} \\ \hline \frac{-2x^{10}y^3 + 4x^8y^4 - 4x^6y^5 + 3x^4y^6}{-2x^{10}y^3 + 2x^8y^4 - 2x^6y^5 + x^4y^6} \\ \hline \frac{2x^6y^5 - 2x^4y^6 + 2x^2y^7 - 2x^2y^3 + y^4}{2x^6y^5 - 2x^4y^6 + 2x^2y^7 + y^4} \end{array}$$

$$\begin{aligned}
 55. \quad & \frac{\alpha^8 - 4\alpha^7b + 4\alpha^6b^2 + 6\alpha^5b^3 - 14\alpha^4b^4 + 4\alpha^3b^5 + 9\alpha^2b^6 - 6\alpha b^7 + b^8}{\alpha^8} (\alpha^4 - 2\alpha^3b + 8ab^3 - b^4) \\
 & \frac{2\alpha^4 - 2\alpha^3b}{-4\alpha^7b + 4\alpha^6b^2} \left| \frac{6\alpha^5b^3 - 14\alpha^4b^4 + 4\alpha^3b^5 + 9\alpha^2b^6}{6\alpha^5b^3 - 12\alpha^4b^4} + 9\alpha^2b^6 \right| \\
 & \frac{2\alpha^4 - 4\alpha^3b + 8ab^3}{-4\alpha^7b + 4\alpha^6b^2} \left| \frac{6\alpha^5b^3 - 14\alpha^4b^4 + 4\alpha^3b^5 + 9\alpha^2b^6}{6\alpha^5b^3 - 12\alpha^4b^4} + 9\alpha^2b^6 \right| \\
 & \frac{2\alpha^4 - 4\alpha^3b + 8ab^3}{-4\alpha^7b + 4\alpha^6b^2} \left| \frac{6\alpha^5b^3 - 14\alpha^4b^4 + 4\alpha^3b^5 + 9\alpha^2b^6}{6\alpha^5b^3 - 12\alpha^4b^4} + 9\alpha^2b^6 \right|
 \end{aligned}$$

$$\begin{aligned}
 56. \quad & \frac{x^{10} - 2x^8y^2 + 5x^6y^4 - 6x^4y^6 + 4x^2y^8 - 12xy^{10} + 9y^{10}}{x^{10}} (x^5 - x^3y^2 + 2xy^4 - 9y^5) \\
 & \frac{2x^5 - x^3y^2}{-2x^8y^2 + 5x^6y^4} \left| \frac{4x^2y^8 - 6x^4y^6 + 4x^2y^8 + 4x^2y^8}{4x^2y^8 - 6x^4y^6 + 4x^2y^8} \right| \\
 & \frac{2x^5 - 2x^3y^2 + 2xy^4}{-2x^8y^2 + 5x^6y^4} \left| \frac{4x^2y^8 - 6x^4y^6 + 4x^2y^8 + 4x^2y^8}{4x^2y^8 - 6x^4y^6 + 4x^2y^8} \right| \\
 & \frac{2x^5 - 2x^3y^2 + 2xy^4}{-2x^8y^2 + 5x^6y^4} \left| \frac{4x^2y^8 - 6x^4y^6 + 4x^2y^8 + 4x^2y^8}{4x^2y^8 - 6x^4y^6 + 4x^2y^8} \right|
 \end{aligned}$$

57.

$$\begin{array}{r}
 a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 (a^2 + 2ab + b^2) \\
 a^4 \overline{) 4a^3b + 6a^2b^2} \\
 4a^3b + 4a^2b^2 \overline{) 2a^2b^3 + 4ab^3 + b^4} \\
 2a^2b^3 + 2a^2b^2 + 4ab^3 + b^4 \overline{) 2a^2b^3 + 4ab^3 + b^4}
 \end{array}$$

$$\begin{array}{r}
 a^2 + 2ab + b^2 (a + b) \\
 a^2 \overline{) 2ab + b^2} \\
 2a + b \overline{) 2ab + b^2}
 \end{array}$$

58.

$$\begin{array}{r}
 16x^4 + 96x^3 + 216x^2 + 216x + 81 (4x^2 + 12x + 9) \\
 16x^4 \overline{) 96x^3 + 216x^2} \\
 96x^3 + 144x^2 \overline{) 72x^2 + 216x + 81} \\
 72x^2 + 216x + 81 \overline{) 72x^2 + 216x + 81}
 \end{array}$$

$$\begin{array}{r}
 4x^2 + 12x + 9 (2x + 3) \\
 4x^2 \overline{) 12x + 9} \\
 4x + 3 \overline{) 12x + 9}
 \end{array}$$

59.

$$\begin{array}{r}
 81x^3 - 108x^2y + 54x^2y^2 - 12x^2y^3 + y^4 (9x^4 - 6x^2y + y^2) \\
 81x^3 \overline{) -108x^2y + 54x^2y^2} \\
 -108x^2y + 36x^2y^2 \overline{) 18x^4y^2 - 12x^2y^3 + y^4} \\
 18x^4y^2 - 12x^2y^3 + y^4 \overline{) 18x^4y^2 - 12x^2y^3 + y^4}
 \end{array}$$

$$\begin{array}{r}
 9x^4 - 6x^2y + y^4 (3x^2 - y) \\
 9x^4 \overline{) -6x^2y + y^4} \\
 -6x^2y + y^4 \overline{) -6x^2y + y^4}
 \end{array}$$

60.

$$\begin{array}{r}
 16a^4 - 32a^3b^2 + 24a^2b^4 - 8ab^6 + b^8 \quad (4a^2 - 4ab^2 + b^4) \\
 \underline{16a^4} \\
 8a^2 - 4ab^2 \quad \begin{array}{l} - 32a^3b^2 + 24a^2b^4 \\ - 32a^3b^2 + 16a^2b^4 \end{array} \\
 \underline{8a^2 - 8ab^2 + b^4} \quad \begin{array}{l} 8a^2b^4 - 8ab^6 + b^8 \\ 8a^2b^4 - 8ab^6 + b^8 \end{array} \\
 \\
 4a^2 - 4ab^2 + b^4 \quad (2a - b^2) \\
 \underline{4a^2} \\
 4a - b^2 \quad \begin{array}{l} - 4ab^2 + b^4 \\ - 4ab^2 + b^4 \end{array}
 \end{array}$$

61.

$$\begin{array}{r}
 16 - 160a + 600a^2 - 1000a^3 + 625a^4 \quad (4 - 20a + 25a^2) \\
 \underline{16} \\
 8 - 20a \quad \begin{array}{l} - 160a + 600a^2 \\ - 160a + 400a^2 \end{array} \\
 \underline{8 - 40a + 25a^2} \quad \begin{array}{l} 200a^2 - 1000a^3 + 625a^4 \\ 200a^2 - 1000a^3 + 625a^4 \end{array} \\
 \\
 4 - 20a + 25a^2 \quad (2 - 5a) \\
 \underline{4} \\
 4 - 5a \quad \begin{array}{l} - 20a^2 + 25a^2 \\ - 20a^2 + 25a^2 \end{array}
 \end{array}$$

62.

$$\begin{array}{r}
 1 - 16x^2y + 96x^4y^2 + 256x^6y^3 + 256x^8y^4 \quad (1 + 8x^2y + 16x^4y^2) \\
 \underline{1} \\
 2 + 8x^2y \quad \begin{array}{l} 16x^2y + 96x^4y^2 \\ 16x^2y + 64x^4y^2 \end{array} \\
 \underline{2 + 16x^2y + 16x^4y^2} \quad \begin{array}{l} 32x^4y^3 + 256x^6y^3 + 256x^8y^4 \\ 32x^4y^3 + 256x^6y^3 + 256x^8y^4 \end{array} \\
 \\
 1 + 8x^2y + 16x^4y^2 \quad (1 + 4x^2y) \\
 \underline{1} \\
 2 + 4x^2y \quad \begin{array}{l} 8x^2y + 16x^4y^2 \\ 8x^2y + 16x^4y^2 \end{array}
 \end{array}$$

$$63. \quad \frac{\alpha^8 - 8\alpha^7b + 28\alpha^6b^2 - 56\alpha^5b^3 + 70\alpha^4b^4 - 56\alpha^3b^5 + 28\alpha^2b^6 - 8\alpha b^7 + b^8}{\alpha^8} (\alpha^8 - 4\alpha^7b + 16\alpha^6b^2 - 4\alpha^5b^3 + 12\alpha^4b^4 - 8\alpha^3b^5 + 12\alpha^2b^6 - 8\alpha b^7 + b^8)$$

$$\frac{2\alpha^4 - 8\alpha^3b + 6\alpha^2b^2}{12\alpha^6b^3 - 48\alpha^5b^4 + 36\alpha^4b^5}$$

$$\frac{2\alpha^4 - 8\alpha^3b + 12\alpha^2b^2 - 4\alpha b^3}{12\alpha^6b^3 - 48\alpha^5b^4 + 36\alpha^4b^5}$$

$$\frac{2\alpha^4 - 8\alpha^3b + 12\alpha^2b^2 - 8\alpha b^3 + b^4}{12\alpha^6b^3 - 48\alpha^5b^4 + 36\alpha^4b^5}$$

$$\frac{2\alpha^4b^5 - 8\alpha^3b^6 + 12\alpha^2b^7 - 8\alpha b^8 + b^9}{12\alpha^6b^3 - 48\alpha^5b^4 + 36\alpha^4b^5}$$

$$\frac{\alpha^4 - 4\alpha^3b + 6\alpha^2b^2 - 4\alpha b^3 + b^4}{\alpha^4} (\alpha^4 - 2\alpha b + b^3)$$

$$\frac{2\alpha^2 - 2\alpha b}{4\alpha^3b + 4\alpha^2b^2}$$

$$\frac{2\alpha^2 - 4\alpha b + b^3}{2\alpha^3b^3 - 4\alpha^2b^4 + b^5}$$

$$\frac{\alpha^2 - 2\alpha b + b^3}{2\alpha - b} \left(\frac{\alpha^2 - 2\alpha b + b^3}{-2\alpha b + b^2} \right)$$

D. A. K.

$$64. \quad x^6 - 16x^5 + 112x^4 - 448x^3 + 1120x^2 - 1792x + 1792x^3 - 1024x + 256(x^4 - 8x^3 + 24x^2 - 32x + 16)$$

$$\frac{2x^4 - 8x^3}{x^3} = \frac{-16x^2 + 112x^3}{-16x^2 + 64x^3}$$

$$2x^4 - 16x^3 + 24x^2 \quad \frac{48x^3 - 448x^2 + 1120x}{48x^3 - 384x^2 + 576x^2}$$

$$2x^4 - 16x^3 + 48x^2 - 32x \quad \frac{-64x^3 + 544x^2 - 1792x + 1792x^3}{-64x^3 + 512x^2 - 1536x^2 + 1024x^2}$$

$$2x^4 - 16x^3 + 48x^2 - 64x + 16 \quad \frac{32x^4 - 256x^3 + 768x^2 - 1024x + 256}{32x^4 - 256x^3 + 768x^2 - 1024x + 256}$$

$$\frac{x^4 - 8x^3 + 24x^2 - 32x + 16}{x^4} (x^3 - 4x + 4)$$

$$2x^2 - 4x \quad \frac{-8x^3 + 24x^2}{-8x^3 + 16x^2}$$

$$2x^2 - 8x + 4 \quad \frac{8x^3 - 32x^2 + 16}{8x^3 - 32x^2 + 16}$$

$$\frac{x^3 - 4x + 4(x - 2)}{x^3}$$

$$2x - 2 \quad \frac{-4x + 4}{-4x + 4}$$

33

EXERCISE VII.

- | | | |
|---|---|--|
| 1. $6'25(25$
4
45 $\overline{225}$
$\overline{225}$ | 2. $2'56(16$
1
26 $\overline{156}$
$\overline{156}$ | 3. $7'29(27$
4
47 $\overline{329}$
$\overline{329}$ |
| 4. $4'41(21$
4
41 $\overline{41}$
$\overline{41}$ | 5. $8'41(29$
4
49 $\overline{441}$
$\overline{441}$ | 6. $12'25(35$
9
65 $\overline{325}$
$\overline{325}$ |
| 7. $22'09(47$
16
87 $\overline{609}$
$\overline{609}$ | 8. $26'01(51$
25
101 $\overline{101}$
$\overline{101}$ | 9. $51'84(72$
49
142 $\overline{284}$
$\overline{284}$ |
| 10. $86'49(93$
81
183 $\overline{549}$
$\overline{549}$ | 11. $34'81(59$
25
109 $\overline{981}$
$\overline{981}$ | 12. $53'29(73$
49
143 $\overline{429}$
$\overline{429}$ |
| 13. $38'44(62$
36
122 $\overline{244}$
$\overline{244}$ | 14. $1'27'69(113$
1
21 $\overline{27}$
$\overline{21}$
223 $\overline{669}$
$\overline{669}$ | 15. $1'48'84(122$
1
22 $\overline{48}$
$\overline{44}$
242 $\overline{484}$
$\overline{484}$ |
| 16. $9'42'49(307$
9
607 $\overline{4249}$
$\overline{4249}$ | 17. $2'99'29(173$
1
27 $\overline{199}$
$\overline{189}$
343 $\overline{1029}$
$\overline{1029}$ | 18. $4'66'56(216$
4
41 $\overline{66}$
$\overline{41}$
426 $\overline{2556}$
$\overline{2556}$ |
| 19. $26'31'69(513$
25
101 $\overline{131}$
$\overline{101}$
1023 $\overline{3069}$
$\overline{3069}$ | 20. $38'31'61(619$
36
121 $\overline{231}$
$\overline{121}$
1229 $\overline{11061}$
$\overline{11061}$ | 21. $49'56'16(70$
49
1404 $\overline{5616}$
$\overline{5616}$ |
| 22. $65'93'44(812$
64
161 $\overline{193}$
$\overline{161}$
1622 $\overline{3244}$
$\overline{3244}$ | 23. $94'28'41(971$
81
187 $\overline{1328}$
$\overline{1309}$
1941 $\overline{1941}$
$\overline{1941}$ | 24. $4'37'64'64(2$
4
409 $\overline{3764}$
$\overline{3681}$
4182 $\overline{8364}$
$\overline{8364}$ |

EXERCISE VII.

35

25. 5'32'22'49 (2307 4 26. 25'62'38'44 (5062 25 27. 49'04'20'09 (7003 49

43 132
129
4607 32249
32249

1006 6238
6036
10122 20244
20244

14003 42009
42009

28. 4'43'52'36 (210-6 4 29. 60'90'24'16 (78-04 49 30. 49'33'65'76 (702-4 49

41 43
41
4206 25236
25236

148 1190
1184
15604 62416
62416

1402 3365
2804
14044 56176
56176

31. 2'25'03'00-01 (1500-1 1 32. 4-44'15'56'25 (2-1075 4

25 125
125
30001 030001
80001

41 44
41
4207 31556
29449
42145 210725
210725

33. 3'32'15'06'25 (18-225 1 34. 18'17'99'90-44 (4263-8 16

28 232
224
362 815
724
3642 9106
7284
36445 182225
182225

82 217
164
846 5399
5076
8523 32390
25569
85268 682144
682144

35. 56'87'57'30-56 (7541-6 49 36. 49-87'18'44 (7-063 49

145 787
725
1504 6257
6016
15081 24130
15081
150826 904956
904956

1406 8718
8436
14122 28244
28244

37. 37°12'46"49 (609·3

36
 1209 $\overline{11248}$
 10881
 12183 $\overline{36549}$
 36549

38. 6°35'19"12'09 (25·208

4
 45 $\overline{235}$
 225
 502 $\overline{1019}$
 1004
 50403 $\overline{151209}$
 151209

39. 2°16'15"17'44"41 (14·7021

1
 24 $\overline{116}$
 96
 287 $\overline{2015}$
 2009
 29402 $\overline{61744}$
 58804
 294041 $\overline{294041}$
 294041

40. 49°42'10"40'60"01 (708001

49
 1403 $\overline{4210}$
 4209
 1406001 $\overline{1406001}$
 1406001

41. ·25 (·5

42. ·00'81 (·09

43. ·00'80 (·08944, &c.

64
 169 $\overline{1600}$
 1521
 1784 $\overline{7600}$
 7136
 17884 $\overline{76400}$
 71536

44. ·60'00'00'00' (·77459...

49
 147 $\overline{1100}$
 1029
 1544 $\overline{7100}$
 6176
 15485 $\overline{92400}$
 77425
 154909 $\overline{1497500}$
 1394181

45.

·00'00'04 (·002

46. ·00'00'40 (·00632...

36
 123 $\overline{400}$
 369
 1262 $\overline{8100}$
 2524

47. ·08'00'00' (·28284...

4
 48 $\overline{400}$
 384
 562 $\overline{1600}$
 1124
 5648 $\overline{47600}$
 45184
 56564 $\overline{241600}$
 226256

EXERCISE VII.

37

48. $50^{\circ}00'00''$ ($\cdot 70710\dots$)

49
1407 $\overline{10000}$
9849
14141 $\overline{15100}$
14141
141420 $\overline{95900}$

49. $10^{\circ}00'00''$ ($\cdot 31622\dots$)

9
61 $\overline{100}$
61
626 $\overline{3900}$
3756
6322 $\overline{14400}$
12644
63242 $\overline{175600}$
126484

50. $10^{\circ}00'$ ($31\cdot 62277\dots$)

9
61 $\overline{100}$
61
626 $\overline{3900}$
3756
6322 $\overline{14400}$
12644
63242 $\overline{175600}$
126484
632447 $\overline{4911600}$
4427129
6324547 $\overline{48447100}$
44271829

51. $2\cdot 50^{\circ}00'$ ($1\cdot 58113\dots$)

1
25 $\overline{150}$
125
308 $\overline{2500}$
2464
3161 $\overline{3600}$
3161
31621 $\overline{43900}$
31621
316223 $\overline{1227900}$
948669

52. $3^{\circ}25'40''$ ($18\cdot 03884\dots$)

1
28 $\overline{225}$
224
3603 $\overline{14000}$
10809
36068 $\overline{319100}$
288544
360768 $\overline{3055600}$
2886144
3607764 $\overline{16945600}$
14431056

53. $4\cdot 90^{\circ}00'$ ($2\cdot 21359\dots$)

4
42 $\overline{90}$
84
441 $\overline{600}$
441
4423 $\overline{15900}$
13269
44265 $\overline{263100}$
221325
442709 $\overline{4177500}$
3984381

54. $50^{\circ}70'10''$ ($7\cdot 12046\dots$)

49
141 $\overline{170}$
141
1422 $\overline{2910}$
2844
142404 $\overline{660000}$
569616
1424086 $\overline{9038400}$
8544516

55. $81\cdot 25$ ($9\cdot 01387\dots$)

81
1801 $\overline{2500}$
1801
18023 $\overline{69900}$
54069
180268 $\overline{1583100}$
1442164
1802767 $\overline{14093600}$
12619369

$$56. \sqrt{.017} = \sqrt{\frac{16}{900}} = \frac{4}{30} = .1\dot{3}. \quad 57. \sqrt{7\cdot\dot{1}} = \sqrt{\frac{64}{9}} = \frac{8}{3} = 2\cdot\dot{6}.$$

$$58. \sqrt{.00004} = \sqrt{\frac{4}{90000}} = \frac{2}{300} = .00\dot{6}.$$

$$59. \sqrt{.21\dot{7}} = \sqrt{\frac{196}{900}} = \frac{14}{30} = .4\dot{6}.$$

$$60. \sqrt{.00284} = \sqrt{\frac{256}{90000}} = \frac{16}{300} = .05\dot{3}.$$

EXERCISE VIII.

1. $3a - 2b + 5a - 6b - 7a + 2b = a - 6b.$
2. $4x^2 - 7y^2 - 6x^2 - 4y^2 + 4x^2 - 10y^2 = 2x^2 - 21y^2.$
3. $5a + 3b + 2c - 2a + 3b + 2a + 4b - 11c = 5a + 10b - 9c.$
4. $x^2 + 3xy - y^2 + 4xy - 5y^2 - 2x^2 + xy - 5y^2 = -x^2 + 8xy - 11y^2.$
5. $12x^3 - 4y^3 - 5z^3 + 2x^3 - y^3 - z^3 - 2y^3 + x^3 = 15x^3 - 7y^3 - 6z^3.$
6. $2a^2 - 3ab + 2ac - b^2 - c^2 - 11ac + 7a^2 + b^2 = 9a^2 - 3ab - 9ac - c^2.$
7. $4a + 2a - 1 - 7a - 15 - a + 1 + 2a + 15 = 0.$
8. $5x - 2y - 3z - 8x - 4y - 2x + 6y - 3z = -6x.$
9. $4a - 3b - 2c - 2a + 7b - 4d + 3a - 4b - 5a + 4d = -2c.$
10. $11x^2 + 4y^2 - 2xy + 3y^2 + 2x^2 - 3xy - 3x^2 + 5xy = 10x^2 + 7y^2.$
11. $7a - \{3a + 2a - 1\} = 7a - 3a - 2a + 1 = 2a + 1.$
12. $8x - \{6x + 2y - 12x + 3y\} = 8x - 6x - 2y + 12x - 3y = 14x - 5y.$
13. $5x - 2y + 3x - 7y - \{6x - 2x - 5y\}$
 $= 5x - 2y + 3x - 7y - 6x + 2x + 5y = 4x - 4y.$
14. $2a - \{3b - 4a + b\} + \{2a - 6b - b + a\}$
 $= 2a - 3b + 4a - b + 2a - 6b - b + a = 9a - 11b.$
15. $3x^2 - \{4xy - 7x^2 + 2xy\} - \{y^2 - 3x^2 - 2xy - 4y^2\}$
 $= 3x^2 - 4xy + 7x^2 - 2xy - y^2 + 3x^2 + 2xy + 4y^2 = 13x^2 - 4xy + 3y^2.$
16. $2x^3 - x^3 + x - 1 - 3x + 4 + 2x^2 + 12x - 1 + 6x = 3x^2 + 16x + 2.$
17. $x^2 - 3x^2 + 2xy + 4y^2 - x - x^2 + 4 - 4x^2 + 3x - 1 = -7x^2 + 2xy + 2x + 4y^2 + 3.$
18. $7x^2 + 2x^2 - 3x - 9 + \{x^2 - 5x + 7 - x^2 - 3x\} + 12 = 9x^2 - 11x + 10.$

19. $-\{3a-6a+5b+7c\}+2a-3c-8a+4b-2c$
 $=-8a+6a-5b-7c+2a-3c-8a+4b-2c=2a-b-12c.$
20. $7a^3-\{3a^3-2b^2+3c^2+4a^3-5c^3\}+b^3-c^3-8b^2-c^3-9b^3$
 $=7a^3-3a^3+2b^2-8c^2-4a^3+5c^3+b^3-c^3-8b^2-c^3-9b^2=-9b^2.$
21. $3x^2-[4x+5x^2-2x^2-3x]=3x^2-4x-5x^2+2x^2+3x=-x.$
22. $4x-[3x-\{2x-2x+5+4\}]=4x-[8x-2x+2x-5-4]$
 $=4x-8x+2x-2x+5+4=x+9.$
23. $12x^2-[7x^2-5x+4x-2x+1+12]=12x^2-7x^2+5x-4x+2x-1-12$
 $=5x^2+8x-13.$
24. $3a-[12a+5b-3c+4b]-\{2a-3b-2c\}$
 $=3a-12a-5b+3c-4b-2a+3b+2c=-11a-6b+5c.$
25. $4x+[7y-2x-5+x]-[4x+3y-2x+7]$
 $=4x+7y-2x-5+x-4x-3y+2x-7=x+4y-12.$
26. $2x^3-4x^3-[3x-\{5x-2x^2-7\}]-2x^3+6x^2-2x$
 $=2x^3-4x^3-[3x-5x+2x^2+7]-2x^3+6x^2-2x$
 $=2x^3-4x^3-3x+5x-2x^2-7-2x^3+6x^2-2x=-7.$
27. $4a-2b-[3a-2b+2a-2c-4a+b+c]$
 $=4a-2b-3a+2b-2a+2c+4a-b-c=3a-b+c.$
28. $7x^2-[2x-\{2x^2-3x+4\}-3x]+5x+3x+7$
 $=7x^2-[2x-2x^2+3x-4-3x]+5x+3x+7$
 $=7x^2-2x+2x^2-3x+4+3x+5x+3x+7=9x^2+6x+11.$
29. $-4a^2+b^2+2a^2-b^2+3c^2+4ab+4a^2-\{3b^2-2c^2+bc\}-ab$
 $=-4a^2+b^2+2a^2-b^2+3c^2+4ab+4a^2-3b^2+2c^2-bc-ab$
 $=2a^2+3ab-3b^2-bc+5c^2.$
30. $5x^2-[x^2-\{2y^2-xy+2x^2\}+3y^2]+2x^2+5xy-y^2+x^2$
 $=5x^2-[x^2-2y^2+xy-2x^2+3y^2]+2x^2+5xy-y^2+x^2$
 $=5x^2-x^2+2y^2-xy+2x^2-3y^2+2x^2+5xy-y^2+x^2$
 $=6x^2+4xy-2y^2+3x^2.$
31. $4a-\{3a-(2a-6a-1)\}-(4a-5a+2)$
 $=4a-\{3a-2a+6a+1\}-4a+5a-2$
 $=4a-3a+2a-6a-1-4a+5a-2=-2a-3.$
32. $7a^2-\{5a^2-2a+2a^2-7a+5\}+3a^2-4a+12$
 $=7a^2-5a^2+2a-2a^2+7a-5+3a^2-4a+12=3a^2+5a+7.$

33. $5a^3 - 2a + 7 - \{3a^3 - 2a - 5a + 6\} - 2a^2 + 7$
 $= 5a^3 - 2a + 7 - 3a^3 + 2a + 5a - 6 - 2a^2 + 7 = 5a + 8.$
34. $2x - [3y - 2x + \{4x - (5y - 3x + 2z)\} + 5z]$
 $= 2x - [3y - 2x + 4x - 5y + 3x - 2z + 5z]$
 $= 2x - 3y + 2x - 4x + 5y - 3x + 2z - 5z = -5x + 2y - z.$
35. $a - [2b + \{3c - (3a - a + b - c)\} + 2b - 3c]$
 $= a - [2b + 3c - 3a + a - b + c + 2b - 3c]$
 $= a - 2b - 3c + 3a - a + b - c - 2b + 3c = 3a - 3b - c.$
36. $3x^2 - [4x - 5 - \{2x^2 - 7x - 2 - 4x^2 + 2x - 7\}]$
 $= 3x^2 - [4x - 5 - 2x^2 + 7x + 2 + 4x^2 - 2x + 7]$
 $= 3x^2 - 4x + 5 + 2x^2 - 7x - 2 - 4x^2 + 2x - 7 = x^2 - 9x - 4.$
37. $5a - 7b - 4c - [6a - 3b - 2c + 4c - \{2a - b + 2a - c\}]$
 $= 5a - 7b - 4c - [6a - 3b - 2c + 4c - 2a + b - 2a + c]$
 $= 5a - 7b - 4c - 6a + 3b + 2c - 4c + 2a - b + 2a - c = 3a - 5b - 7c.$
38. $8x^2 - 5xy + 6y^2 - [2x^2 - \{3xy - y^2 + 3x^2 - 2xy - 5y^2\}]$
 $= 8x^2 - 5xy + 6y^2 - 2x^2 + 3xy - y^2 + 3x^2 - 2xy - 5y^2 = 9x^2 - 4xy.$
39. $-x^2 - 2ax + 3a^2 + [a^2 - x^2 - \{2ax - a^2 + 7ax + 5x^2\} - 2ax]$
 $= -x^2 - 2ax + 3a^2 + a^2 - x^2 - 2ax + a^2 - 7ax - 5x^2 - 2ax$
 $= -7x^2 - 13ax + 5a^2.$
40. $11a^3 - 2a^2 + 3a - [7a - 2 + 9a^3 + 12a^2 - 11a + 5 - a^3]$
 $= 11a^3 - 2a^2 + 3a - 7a + 2 - 9a^3 - 12a^2 + 11a - 5 + a^3$
 $= 3a^3 - 14a^2 + 7a - 3.$
41. $11x - 7 + 8x - 6 + 7x - 14 = 26x - 27.$
42. $5x^3 - 3 - 8x + 1 - 8x^2 - 10x + 6 = -3x^2 - 18x + 4.$
43. $4a - 22a - 11b + 33c + 21a - 7b = 3a - 18b + 33c = 3(a - 6b + 11c).$
44. $6a^3 - 12a + 30 - 2a^3 - 2a^2 - 2 - a^3 + a^2 - 2a + 3 = -3a^3 + 5a^2 - 14a + 31.$
45. $4x^2 - 12x + 8 - 6x^2 - 12 - 35x^2 + 15x + 60 = -37x^2 + 3x + 56.$
46. $5x^2 - 3x + x^2 - 4 + 8x^2 - 2x + 10 + 15 = 14x^2 - 5x + 21.$
47. $2a^2 - 3a^2 - a + 2a^2 - 10a + 12 - 3a^2 + 6a + 3 = -2a^2 - 5a + 15.$
48. $6x^2 + 4xy - 4xz + 3xy - 3xz + 3y^2 - 3x^2 + 6y^2 = 3x^2 + 7xy - 7xz + 9y^2.$
49. $4a^2 - 4ab - 3a^2 + 6ab + 3b^2 + 2a^2 - 2ab = 3a^2 + 3b^2 = 3(a^2 + b^2).$
50. $a^3 - 2a^2 + 10a^3 - 30a^2 - 5(a^3 - a^2 + 2a)$
 $= a^3 - 2a^2 + 10a^3 - 30a^2 - 5a^3 + 5a^2 - 10a$
 $= 6a^3 - 27a^2 - 10a = a(6a^2 - 27a - 10).$

51. $a^2 + ab - ac - 3ab + 3b^2 + 3bc + 2ac + 2bc + 4c^2$
 $= a^3 - 2ab + ac + 3b^2 + 5bc + 4c^2.$
52. $3x^2 + 2xy - xz - 3xy - 3y^2 - 12yz + 12xz - 6yz - 6z^2$
 $= 3x^2 - xy + 11xz - 3y^2 - 18yz - 6z^2.$
53. $7x^2 + 7xy + 7y^2 - 12xy + 24xz + 3xy + 6yz = 7x^2 - 2xy + 7y^2 + 24xz + 6yz.$
54. $2(x^2 + 3x^2 - 3yz) - 5xy + 10y^2 + 7(y^2 - y^2 + yz)$
 $= 2x^2 + 6x^2 - 6yz - 5xy + 10y^2 + 7yz = 8x^2 - 5xy + 10y^2 + yz.$
55. $3a^2 - [2a^2 - 2a^2 + 2a - 2 + 2]$
 $= 3a^2 - 2a^2 + 2a^2 - 2a + 2 - 2 = 3a^2 - 2a = a(3a - 2).$
56. $3[4x - 5 - 6x + 8 + 5\{2x - 3 - 2x + 7x - 35\}]$
 $= 3[4x - 5 - 6x + 8 + 10x - 15 - 10x + 35x - 175]$
 $= 3[33x - 187] = 33(3x - 17).$
57. $3a^2 - 3b^2 - [2a^2 - 2b^2 - 2ab - 2b^2 + 2ab]$
 $= 3a^2 - 3b^2 - 2a^2 + 2b^2 + 2ab + 2b^2 - 2ab = a^2 + b^2.$
58. $x - [2x - x + 6x + 3y - 6x - 3y] = x - 2x + x - 6x - 3y + 6x + 3y = 0.$
59. $7a^4 - 7a^2 + 14a + 2a^3 - 2a^2 + 8a - 3a^3 + 3a^2 + 3a^4 - 6a^2 + 3a^3$
 $= 10a^4 + 2a^3 - 12a^2 + 22a.$
60. $-4x^3 + 4x^2y - 4x^2z + 8x^2y - 3xy^2 + 3xyz - 2x^2y + 6xy^2 - 6y^3$
 $= -4x^3 + 5x^2y + 3xy^2 - 4x^2z + 3xyz - 6y^3.$

EXERCISE IX.

- | | |
|---|--|
| 1. $9x - 8x = 19 - 15;$
$x = 4.$ | 2. $28x - 27x = 50 - 17;$
$x = 33.$ |
| 3. $31x - 30x = 89 - 41;$
$x = 48.$ | 4. $12x - 11x = 17 - 4;$
$x = 13.$ |
| 5. $63x - 62x = 80 - 7;$
$x = 73.$ | 6. $-7x + 8x = 100 - 50;$
$x = 50.$ |
| 7. $2x - x = 7 - 4;$
$x = 3.$ | 8. $13x - 12x = 58 + 41;$
$x = 99.$ |
| 9. $-49x + 50x = 17 - 16;$
$x = 1.$ | 10. $67x - 66x = 2000 - 1328;$
$x = 672.$ |
| 11. $12x - 9x = 13 - 7;$
$3x = 6; \therefore x = 2.$ | 12. $51x - 44x = 14 - 7;$
$7x = 7; \therefore x = 1.$ |

13. $13x - 9x = 27 - 11$;
 $4x = 16$; $\therefore x = 4$.
14. $14x - 8x = 4 + 20$;
 $6x = 24$; $\therefore x = 4$.
15. $17x - 2x = 50 - 5$;
 $15x = 45$; $\therefore x = 3$.
16. $13x + 7x = 19 + 21$;
 $20x = 40$; $\therefore x = 2$.
17. $-7x + 20x = 100 - 48$;
 $13x = 52$; $\therefore x = 4$.
18. $13x - 8x = 48 - 23$;
 $5x = 25$; $\therefore x = 5$.
19. $4x - x = 60 - 57$;
 $3x = 3$; $\therefore x = 1$.
20. $-2x + 17x = 135 - 90$;
 $15x = 45$; $\therefore x = 3$.
21. $15x - 14x = 21 - 37$;
 $x = -16$.
22. $24x - 23x = 3 - 36$;
 $x = -33$.
23. $15x - 13x = 18 - 24$;
 $2x = -6$; $\therefore x = -3$.
24. $17x - 12x = 20 - 40$;
 $5x = -20$; $\therefore x = -4$.
25. $3x - 4x = 20 - 15$;
 $-x = 5$; $\therefore x = -5$.
26. $7x - 8x = 20 + 8$;
 $-x = 28$; $\therefore x = -28$.
27. $14x - 18x = 29 - 17$;
 $-4x = 12$; $\therefore x = -3$.
28. $-7x - 2x = 57 - 21$;
 $-9x = 36$; $\therefore x = -4$.
29. $-9x - 8x = 51 - 34$;
 $-17x = 17$; $\therefore x = -1$.
30. $4x - 10x = 21 + 15$;
 $-6x = 36$; $\therefore x = -6$.
31. $15x - 13x = 20 - 11$;
 $2x = 9$; $\therefore x = \frac{9}{2} = 4\frac{1}{2}$.
32. $7x - 4x = 20 + 15$;
 $3x = 35$; $\therefore x = \frac{35}{3} = 11\frac{2}{3}$.
33. $30x - 25x = 18 - 12$;
 $5x = 6$; $\therefore x = \frac{6}{5} = 1\frac{1}{5}$.
34. $29x + 11x = 19 + 11$;
 $40x = 30$; $\therefore x = \frac{30}{40} = \frac{3}{4}$.
35. $7x + 8x = 12 - 15$;
 $15x = -3$; $\therefore x = -\frac{1}{5}$.
36. $2x - 5x = -17 + 9$;
 $-3x = -8$; $\therefore x = \frac{8}{3} = 2\frac{2}{3}$.
37. $15x - 12x = 11 - 11$;
 $3x = 0$; $\therefore x = 0$.
38. $16x - 12x = -31 + 17$;
 $4x = -14$; $\therefore x = -3\frac{1}{2}$.
39. $13x - 15x = 19 + 70$;
 $-2x = 89$; $\therefore x = -44\frac{1}{2}$.
40. $77x - 44x = 28 - 50$;
 $33x = -22$; $\therefore x = -\frac{22}{33} = -\frac{2}{3}$.
41. $12x + 15x - 11x = 8 + 28 - 4$;
 $16x = 32$; $\therefore x = 2$.
42. $13x - 6x = 5 + 12 + 11$;
 $7x = 28$; $\therefore x = 4$.

43. $15x - 12x + 2x = 19 + 32 - 51$; $5x = 0$; $\therefore x = 0$. 44. $21x + 15x + 11x = 47 + 26 + 14 + 7$; $47x = 94$; $\therefore x = 2$.
45. $42x - 11x + 13x = 121 + 23 - 100$; $44x = 44$; $\therefore x = 1$.
46. $3x - 2x + 11x = 19 + 7 - 4$; $12x = 16$; $\therefore x = \frac{4}{3} = 1\frac{1}{3}$.
47. $7x - 3x + 12x = 4 + 10$; $16x = 14$; $\therefore x = \frac{7}{8}$.
48. $29x - 13x - 7x = 81 + 20 - 56 + 17$; $9x = 12$; $\therefore x = \frac{12}{9} = 1\frac{1}{3}$.
49. $14x - 9x - 4x + 7x = 13 + 93 - 70$; $8x = 36$; $\therefore x = 4\frac{1}{2}$.
50. $23x - 2x + 11x = 56 + 24 + 41 - 17$; $32x = 104$; $\therefore x = 3\frac{1}{4}$.
51. $9x - 3x + 7x + x = 25 - 20$; $14x = 5$; $\therefore x = \frac{5}{14}$.
52. $17x + 5x - 7x = 12 + 21 + 2$; $15x = 35$; $\therefore x = \frac{35}{15} = 2\frac{1}{3}$.
53. $43x - 15x - 10x = -17 + 35$; $18x = 18$; $\therefore x = 1$.
54. $15x - 7x - 3x = -15 + 10$; $5x = -5$; $\therefore x = -1$.
55. $12x + 15x - 21x + x = 36 - 4$; $7x = 32$; $\therefore x = 4\frac{2}{7}$.
56. $9x - x = -14 - 13 - 7$; $8x = -34$; $\therefore x = -4\frac{1}{4}$.
57. $13x + 15x - x = -10 - 97 + 17$; $27x = -90$; $\therefore x = -\frac{90}{27} = -3\frac{1}{3}$.
58. $x - 15x + 11x = -60 + 14 + 12$; $-3x = -34$; $\therefore x = 11\frac{1}{3}$.

59. $64x - 11x - 13x - 19x = -27$;
 $21x = -27$; $\therefore x = -1\frac{1}{3}$.
60. $-18x = 2 - 15 + 67 - 12 + 21$;
 $-18x = 63$; $\therefore x = -\frac{63}{18} = -3\frac{1}{2}$.
61. $5x - 35 + 2x - 18 = 17$;
 $5x + 2x = 17 + 35 + 18$;
 $7x = 70$; $\therefore x = 10$.
62. $7x - 28 + 6x - 3 = 34$;
 $7x + 6x = 34 + 28 + 3$;
 $13x = 65$; $\therefore x = 5$.
63. $9x - 3 + 10x - 30 = 7x + 15$;
 $9x + 10x - 7x = 15 + 3 + 30$;
 $12x = 48$; $\therefore x = 4$.
64. $x - 4 - 6x + 27 = 2x - 6 - 41$;
 $x - 6x - 2x = -6 - 41 + 4 - 27$;
 $-7x = -70$; $\therefore x = 10$.
65. $24x - 12 - 12x + 9 - 7x + 7 = 9$;
 $24x - 12x - 7x = 9 + 12 - 9 - 7$;
 $5x = 5$; $\therefore x = 1$.
66. $6x - 3 - 24x + 20 = 48x - 60 - 22$;
 $6x - 24x - 48x = -60 - 22 + 3 - 20$;
 $-66x = -99$; $\therefore x = \frac{99}{66} = \frac{3}{2} = 1\frac{1}{2}$.
67. $4x - 12 - 10x + 15 = 12 - x - 9$;
 $4x - 10x + x = 12 - 9 + 12 - 15$;
 $-5x = 0$; $\therefore x = 0$.
68. $14x - 21 - 2x + 4 = 3x - 10x + 18 + 3$;
 $14x - 2x - 3x + 10x = 18 + 3 + 21 - 4$;
 $19x = 38$; $\therefore x = 2$.
69. $20x - 90 - 28x + 133 + 5 = 4x - 6x + 9$;
 $20x - 28x - 4x + 6x = 9 + 90 - 133 - 5$;
 $-6x = -39$; $\therefore x = \frac{39}{6} = 6\frac{1}{2}$.
70. $5x + 21x - 12 - 20x + 14 = 4 - x + 5$;
 $5x + 21x - 20x + x = 4 + 5 + 12 - 14$;
 $7x = 7$; $\therefore x = 1$.

71. The L. C. D. is 12: hence $6x+4x+3x=156$;
 $13x=156$; $\therefore x=12$.
72. L. C. D. is 24: hence $3x+8x+4x=360$;
 $15x=360$; $\therefore x=24$.
73. L. C. D., 50: hence $5x+10x=650+2x$;
 $5x+10x-2x=650$;
 $13x=650$; $\therefore x=50$.
74. L. C. D., 12: hence $4x+3x+6x=12x+96$;
 $4x+3x+6x-12x=96$; $\therefore x=96$.
75. L. C. D., 20: hence $10x-4x+5x=20x-180$;
 $10x-4x+5x-20x=-180$;
 $-9x=-180$; $\therefore x=20$.
76. L. C. D., 24: hence $3x-6x+8x=12x-168$;
 $3x-6x+8x-12x=-168$;
 $-7x=-168$; $\therefore x=24$.
77. L. C. D., 8: hence $6x+8x=7x+16x-72$;
 $6x+8x-7x-16x=-72$;
 $-9x=-72$; $\therefore x=8$.
78. L. C. D., 21: hence $15x+63x=1324+14x$;
 $64x=1324$; $x=21$.
79. L. C. D., 6: hence $4x-5x=6x-3x+2$;
 $4x-5x-6x+3x=2$;
 $-4x=2$; $\therefore x=-\frac{2}{4}=-\frac{1}{2}$.
80. L. C. D., 60: hence $-42x+90+45x=170-32x-80$;
 $-42x+45x+32x=170-80-90$;
 $85x=0$; $\therefore x=0$.
81. L. C. D., 20: hence $25x-40+4x-8=300$;
 $29x=348$; $\therefore x=12$.
82. L. C. D., 60: hence $40x+25=420+9x-12$;
 $31x=383$; $\therefore x=12\frac{1}{31}$.
83. L. C. D., 24: hence $12x-20+12x-21=56$;
 $24x=97$; $\therefore x=4\frac{1}{24}$.
84. L. C. D., 6: hence $4x+14-3x+15=36$; $\therefore x=7$.
85. L. C. D., 10: hence $12x+14-2x+1=45$;
 $10x=30$; $\therefore x=3$.

86. $24x - 27 - 6x + 10 + 23x - 98 = 0$; (L. C. D., 42);
 $46x = 115$; $\therefore x = \frac{115}{46} = \frac{5}{2} = 2\frac{1}{2}$.
87. $35x + 10 - 16x - 12 = 228$; (L. C. D., 60);
 $19x = 230$; $\therefore x = 12\frac{1}{5}$.
88. $15x - 30 - 24x + 26 = 96x + 8x - 28$; (L. C. D., 24);
 $-113x = -24$; $\therefore x = \frac{24}{113}$.
89. $42(4x - 1) - 9(3x + 2) = 378 + 7(5x - 2)$; (L. C. D., 63);
 $168x - 42 - 27x - 18 = 378 + 35x - 14$;
 $106x = 424$; $\therefore x = 4$.
90. $6(4x - 11) + 3(3x - 4) = 40 - (3x + 13)$; (L. C. D., 12);
 $24x - 66 + 9x - 12 = 40 - 3x - 13$;
 $36x = 105$; $\therefore x = \frac{105}{36} = 2\frac{1}{4}$.
91. $10x + 15 - 33x + 77 + 55x = 220$; (L. C. D., 55);
 $32x = 128$; $\therefore x = 4$.
92. $10x - 25 - 9x - 27 - 15x = 90x - 780$; (L. C. D., 45);
 $-104x = -728$; $\therefore x = 7$.
93. $10x - 14 + 2x - 11 - 3x = 18x - 25$; (L. C. D., 6);
 $-9x = -25 + 14 + 11 = 0$; $\therefore x = 0$.
94. $36x - 48 + 9x + 15 - 28 + 16x = 0$; (L. C. D., 12);
 $61x = 61$; $\therefore x = 1$.
95. $14x - 28 - 6x + 21 = 21x + 84$; (L. C. D., 42);
 $-13x = 91$; $\therefore x = -7$.
96. $30x - 48 - 40x + 28 + 24 = 15x - 16$; (L. C. D., 12);
 $-25x = -20$; $\therefore x = \frac{20}{25} = \frac{4}{5}$.
97. $4x + 22 - 25 + 30x = 70x + 15$; (L. C. D., 10);
 $-36x = 18$; $\therefore x = -\frac{18}{36} = -\frac{1}{2}$.
98. $90x - 25x + 10 + 150x + 255 + 423 = 0$; (L. C. D., 45);
 $215x = -688$; $\therefore x = -\frac{688}{215} = -3\frac{1}{5}$.
99. $20x + 32x + 24 + 30x + 15 = 24x - 222$; (L. C. D., 20);
 $58x = -261$; $\therefore x = -\frac{261}{58} = -4\frac{1}{2}$.

100. $105 + 140x - 80x - 40 = 42x - 105$;
 $68x = -170$; $x = -\frac{170}{68} = -2\frac{1}{2}$.
101. $72x - 756 - 56x - 42 + 278 = 42x - 525$;
 $-26x = 0$; $\therefore x = 0$.
102. $21x + 85 - 176x - 48 = 42x - 210$;
 $-197x = -197$; $\therefore x = 1$.
103. $270x - 24x - 12 - 100x - 175 = 105$;
 $146x = 292$; $\therefore x = 2$.
104. $56x - 14 + 12x - 60 = 63x - 84$;
 $5x = -10$; $\therefore x = -2$.
105. $4x - 24 - 6x + 2x - 16 = 65 - 15x$;
 $15x = 105$; $\therefore x = 7$.
106. $6x + 8 = 10x - 15x - 60 + 45 - 1 + 3x$;
 $8x = -24$; $\therefore x = -3$.
107. $16x - 16 - 8x - 24 = 5x + 35x - 280 + 70x$;
 $-102x = -240$; $\therefore x = 2\frac{4}{3}$.
108. $4x - 15x + 24x + 12 = 12x + 10$; $x = -2$.
109. $16x + 56 - 18x - 60 = x + 5$; (L. C. D., 2);
 $-3x = 9$; $\therefore x = -3$.
110. $105x - 175 - 60x + 90 = 3x - 1$; (L. C. D., 5);
 $42x = 84$; $\therefore x = 2$.
111. $36x + 54 - 36x - 45 = 1 - 6x$; (L. C. D., 3);
 $6x = -8$; $\therefore x = -1\frac{1}{3}$.
112. $12x + 4 - 2x - 5 = 15x - \frac{3x+9}{2}$;
 $24x + 8 - 4x - 10 = 30x - 3x - 9$; (L. C. D., 2);
 $-7x = -7$; $\therefore x = 1$.
113. $120x - 180 - 10x - 70 + 12x - 4 = 5x - 20$; (L. C. D., 20);
 $117x = 234$; $\therefore x = 2$.
114. $72x - 72 + 27x - 45 - 16x + 12 + 18x - 198 = 0$; (L. C. D., 36);
 $101x = 303$; $\therefore x = 3$.
115. $7x - 7 + 10x - 15 - 105 = 70x + 350$;
 $-53x = 477$; $\therefore x = -9$.
116. $135x + 27 - 56x + 16 - 24x = 6x - 6$; (L. C. D., 36);
 $49x = -49$; $\therefore x = -1$.

EXERCISE IX.

117. $90x + 30 - 6x - 24 + 300 + 10x + 70 = 0$; (L. C. D., 15);
 $94x = -376$; $\therefore x = -4$.
118. $150x + 15x - 20 - 72x + 12 - 120 - 60x + 150 = 0$;
 $33x = -22$; $\therefore x = -\frac{2}{3}$.
119. $240x + 1440 - 20x - 24x - 240 = 78x + 390 - 75$;
 $118x = -885$; $\therefore x = -\frac{885}{118} = -7\frac{1}{2}$.
120. $-5(x+2) - \frac{x+2}{3} + \frac{30x}{7} + \frac{55}{7} + \frac{13}{5} = 0$;
 $-525x - 1050 - 35x - 70 + 450x + 825 + 273 = 0$;
 $-110x = 22$; $\therefore x = -\frac{22}{110} = -\frac{1}{5}$.

EXERCISE X.

1. Let x be the required number;
 then $x + 12 = 4x$;
 $\therefore x - 4x = -12$;
 $-3x = -12$; $\therefore x = 4$.
2. Let x be the required number;
 then $2x - 64 = 16$;
 $2x = 16 + 64 = 80$; $\therefore x = 40$.
3. Let x be the sum of money, in shillings;
 then $x + 36 = 3x$;
 $x - 3x = -36$;
 $-2x = -36$; $\therefore x = 18$.
4. Let x be the number added;
 then $2500 + x = 11x$;
 $x - 11x = -2500$;
 $-10x = -2500$; $\therefore x = 250$.
5. Let x be the required number;
 then $\frac{x}{3} + \frac{x}{5} = 64$;
 $5x + 3x = 960$;
 $8x = 960$; $\therefore x = 120$.

6. Let
- x
- be the number;

then

$$\frac{x}{4} + \frac{x}{10} = 35;$$

$$5x + 2x = 700;$$

$$7x = 700; \therefore x = 100.$$

7. Let
- x
- be the number;

then

$$\frac{x}{3} - \frac{x}{7} = 44;$$

$$7x - 3x = 924;$$

$$4x = 924; \therefore x = 231.$$

8. Let
- x
- be the number;

then

$$\frac{x}{10} + 4 = \frac{x}{5};$$

$$x + 40 = 2x; \therefore x = 40.$$

9. Let
- x
- be the son's age, in years;

then $3x$ is the father's

$$\therefore 3x + x = 48; \therefore x = 12.$$

10. Let
- x
- be the price of the carriage, in £s;

then $3x$ is horse ...

$$\therefore 3x + x = 84; \therefore x = 21;$$

i.e., the carriage cost £21, and the horse £63.

11. Let
- x
- be the number of miles between
- B
- and
- C
- ;

then $4x$ is A and B .

$$\therefore 4x - x = 51;$$

$$3x = 51; \therefore x = 17;$$

hence distance from A to B is 68 miles.

12. Let
- x
- be the number of boys in smaller school;

then $3x$ is larger

$$\therefore 3x - x = 450;$$

$$2x = 450; \therefore x = 225;$$

hence 225 and 675 are the numbers.

13. Let
- x
- be
- B
- 's contribution in £s;

then $x + 15$ is A 's

$$\therefore x + x + 15 = 100;$$

$$2x = 85; \therefore x = 42\frac{1}{2};$$

 B gave £42. 10s.; A gave £57. 10s.

D. A. K.

14. Let x be the number of shillings of the younger boy;
then $x+8$ is older ... ;

$$\therefore x+x+8=50;$$

$$2x=42; \therefore x=21;$$

hence £1. 1s., and £1. 9s. are the sums,

15. Let x be the horse-power of the smaller;
then $x+16$ is larger;

$$x+x+16=80;$$

$$2x=64; \therefore x=32;$$

hence 32 and 48 are the powers,

16. Let x be A 's contribution in £s;
then $x+10$ is B 's ;

$x+35$ is C 's ;

then $x+x+10+x+35=150;$

$$3x=105; \therefore x=35;$$

hence £35, £45, £70 are the contributions.

17. Let x be the share of C , in £s;
then $x+50$ is B 's share;

$x+150$ is A 's share:

hence $x+x+50+x+150=1000;$

$$3x=800; \therefore x=266\frac{2}{3}.$$

18. Let x be one of the smaller shares, in £s;
then $x+2000$ is the large share:

hence $x+x+x+2000=20000;$

$$3x=18000; \therefore x=6000;$$

the shares are £6000, £6000, £8000.

19. Let x be the number;

then $\frac{x}{4} - \frac{x}{5} = 7;$

$$5x-4x=140; \therefore x=140.$$

20. Let x be the number;

then $\frac{x}{5} = \frac{x}{12} + 28;$

$$12x=5x+1680;$$

$$7x=1680; \therefore x=240.$$

21. Let x be the number;

then $4(x+16)=10x+22;$

$$4x-10x=22-64;$$

$$-6x=-42; \therefore x=7.$$

22. Let x be one part;
then $40 - x$ is the other part;

$$\begin{aligned}\therefore x &= \frac{3}{5}(40 - x); \\ 5x &= 120 - 3x; \\ 8x &= 120; \therefore x = 15;\end{aligned}$$

hence 15 and 25 are the parts.

23. Let x be one part;
then $60 - x$ is the other part;

$$\begin{aligned}\therefore \frac{1}{3}x &= \frac{1}{2}(60 - x); \\ 2x &= 180 - 3x; \\ 5x &= 180; \therefore x = 36;\end{aligned}$$

36 and 24 are the parts.

24. Let x be number of shillings in one part;
then $21 - x$ is the other part;

$$\begin{aligned}\therefore x &= \frac{8}{4}(21 - x); \\ 4x &= 63 - 3x; \therefore x = 9;\end{aligned}$$

9s. and 12s. are the parts.

25. Let x be the greater part,
then $81 - x$ is the less;

$$\begin{aligned}\therefore \frac{1}{18}x &= 2(81 - x); \\ x &= 2106 - 26x; \therefore x = 78;\end{aligned}$$

78 and 3 are the parts.

26. Let x be the greater part;
then $192 - x$ is the less;

$$\begin{aligned}\therefore \frac{1}{2}x &= \frac{1}{3}(192 - x) + 21; \\ 3x &= 384 - 2x + 126; \\ 5x &= 510; \therefore x = 102;\end{aligned}$$

102 and 90 are the parts.

27. Let x be the number;

$$\begin{aligned}\text{then} \quad 2x - 86 &= \frac{4}{5}x; \\ 10x - 180 &= 4x; \therefore x = 30.\end{aligned}$$

28. Let x be the number of years;
 then x years hence, the father will be $42+x$,
 the son $6+x$;
 hence $42+x=3(6+x)$;
 $\therefore x-8x=18-42$; $\therefore x=12$.

29. Let x be the number of years;
 then x years ago, A was $72-x$;
 B ... $48-x$;
 $\therefore 72-x=5(48-x)$;
 $-x+5x=240-72$; $\therefore x=42$.

30. Let x be the number of years;
 then $70-x=\frac{7}{2}(35-x)$;
 $140-2x=245-7x$;
 $5x=105$; $\therefore x=21$.

31. Let x be the number of years;
 then $42-x=\frac{2}{3}(56-x)$;
 $126-3x=112-2x$; $\therefore x=14$.

32. Let x be the shillings which A gives B ;
 then $48+x=3(60-x)$;
 $4x=132$; $\therefore x=33$.

33. Let x be the son's age, in years;
 then $4x$ is the father's age;
 $\therefore 4x-12=16(x-12)$;
 $12x=180$; $\therefore x=15$;
 hence the ages are 60 and 15.

34. Let x be B 's money in shillings;
 then $5x$ is A 's;
 $\therefore 5x-23=2(x+23)$;
 $3x=69$; $\therefore x=23$;
 hence A had $115s.=£5. 15s$.

35. Let x be the number of £s;
 then $20+x=3(15-x)$;
 $4x=25$; $\therefore x=6\frac{1}{4}=£6. 5s$.

36. Let x be C 's money in £s;
 then $x+20$ is B 's money;
 and $2x+40$ is A 's

$$\therefore 2x+40+x+20+x=300;$$

$$4x=240; \therefore x=60;$$

hence the sums are £60, £80, and £160.

37. Let x be A 's money in £s;
 then $x+50$ is B 's money,
 and $x+185$ is C 's

$$\therefore x+x+50+x+185=760;$$

$$3x=525; \therefore x=175;$$

hence A has £175, B £225, C £360.

38. Let x be the £s spent by A ;
 then B spends $\frac{1}{2}x$ £s;

and C spends $\frac{3}{4}$ of $\frac{3}{2}x = \frac{9x}{8}$ £s;

$$\therefore x + \frac{1}{2}x + \frac{9x}{8} = 52\frac{1}{2};$$

$$8x + 4x + 9x = 420; \therefore x = 20;$$

hence A spends £20, B £10, C £22, 10s.

39. Let x be the number of £s divided;
 then

$$A \text{ has } \frac{x}{2} - 5;$$

$$B \text{ has } \frac{x}{2} - 8;$$

$$C \text{ has } \frac{x}{2} - 15\frac{1}{2};$$

$$\therefore \frac{x}{2} - 5 + \frac{x}{2} - 8 + \frac{x}{2} - 15\frac{1}{2} = x;$$

$$x - 10 + x - 16 + x - 31 = 2x; \therefore x = 57.$$

40. Let x be the smaller number;
 then $x+60$ is the larger;

$$\therefore \frac{1}{7}(x+60) = \frac{1}{3}x;$$

$$3x + 180 = 7x; \therefore x = 45;$$

45 and 105 are the numbers.

41. Let x be the number;

then $\frac{x}{2} - 100 = 100 - \frac{x}{8};$
 $8x + 2x = 1200; \therefore x = 240.$

42. Let x be the number;

then $4x - 50 = 50 - x; \therefore x = 20.$

43. Let x be the smaller number;

then $x + 20$ is the larger;
 $\therefore \frac{1}{7}(x + 20) = \frac{x}{10} + 5;$
 $10x + 200 = 7x + 350; \therefore x = 50;$

hence 50 and 70 are the numbers.

44. Let x be the smaller number;

then $x + 10$ is the larger;
 $\therefore \frac{1}{2}(x + 10) = 2x + 2;$
 $x + 10 = 4x + 4; \therefore x = 2;$

12 and 2 are the numbers.

45. Let x be the smaller number;

then $x + 8$ is the greater;
 $\therefore 4x = 2(x + 8) + 10; \therefore x = 13;$
 13 and 21 are the numbers.

46. Let x be one part;

then $144 - x$ is the other;
 $\therefore 144 - x + 16 = 2(x - 10);$
 $3x = 180; \therefore x = 60;$

60 and 84 are the numbers.

47. Let x be one part;

$70 - x$ is the other;
 then $\frac{x}{2} + \frac{70 - x}{3} = 31;$

$$3x + 140 - 2x = 186; \therefore x = 46;$$

hence 46 and 24 are the parts.

48. Let x be A 's money, in £s;

then B has $77 - x;$
 and C has $105 - x;$
 $\therefore 77 - x + 105 - x = 112;$
 $2x = 70; \therefore x = 35;$

hence A has £35; B , £42; C , £70.

49. Let x be A 's money in £s;

then B has $49\frac{1}{2} - x$;

and C has $38\frac{1}{2} - x$;

$$\therefore 49\frac{1}{2} - x + 38\frac{1}{2} - x = 52\frac{1}{2};$$

$$197 - 4x + 154 - 4x = 211;$$

$$8x = 140; \therefore x = 17\frac{1}{2};$$

$\therefore A$ has £17. 10s.; B has £31. 15s.; C has £21.

50. Let x be the number;

then $\frac{x}{6} + \frac{x}{8} = \frac{x}{12} + 20$;

$$4x + 3x = 2x + 480; \therefore x = 96.$$

51. Let x be the number;

then $\frac{x+20}{6} = \frac{x}{4}$; $\therefore x = 40$.

52. Let x be the first-class fare, in shillings;

and $x - 5$ the second-class

then $\frac{x}{3} = \frac{x-5}{2}$; $\therefore x = 15$;

hence 15s. and 10s. are the fares.

53. Let x be the number;

then $\frac{x}{6} = \frac{1}{2}(80 - x)$;

$$x = 240 - 3x; \therefore x = 60.$$

54. Let x be the number of panes in each window of upper row;

then $x + 4$ middle row;

and $x + 6$ lowest row;

$$\therefore 6x + 6(x+4) + 6(x+6) = 168;$$

$$18x = 108; \therefore x = 6:$$

$\therefore 6, 10, 12$ are the numbers.

55. Let x be the number of hours for the first part;

then $\frac{3x}{2}$ is second

and $\frac{15x}{8}$ third

$$\therefore x + \frac{3x}{2} + \frac{15x}{8} = 5\frac{1}{2};$$

$$24x + 36x + 45x = 140;$$

$$\therefore x = \frac{140}{105} = 1\frac{1}{3} = 1 \text{ hr. } 20'.$$

56. Let x be A 's income in £s;
then $x + 450$ is B 's

$$\therefore \frac{x}{2} = \frac{1}{6}(x + 450) + 100;$$

$$3x = x + 450 + 600; \therefore x = 525;$$

\therefore £525 and £975 are the incomes.

57. Let x be the sum of money, in £s;
after first transaction, I have $\frac{2x}{3} + 50$;

after second transaction, I have $\frac{3}{4}\left(\frac{2x}{3} + 50\right) + 70$;

$$\therefore \frac{3}{4}\left(\frac{2x}{3} + 50\right) + 70 = 120;$$

$$\frac{x}{2} + 37\frac{1}{2} = 50; \therefore x = 25.$$

58. Let x be the number of persons;
then each receives $\frac{150}{x}$ shillings;

$$\therefore \frac{150}{x} = \frac{150}{x+5} + 5;$$

$$150x + 750 = 150x + 5x^2 + 25x;$$

$$5x^2 + 25x = 750;$$

$$x^2 + 5x = 150; \therefore x = 10.$$

(N.B. This produces a quadratic equation; the question is changed in the 5th edition.)

58 (As in Edition V. 1886). Let x be the sum of money, in shillings;
then

$$\frac{x}{10} = \frac{x}{15} + 5;$$

$$3x = 2x + 150; \therefore x = 150 = \text{£}7. 10s.$$

59. Let x be number of points made by B ;
then

$$\frac{2}{5}x + 3 \text{ was } A\text{'s score};$$

$$\therefore \frac{2}{5}x + 2 = \frac{x}{2};$$

$$4x + 20 = 5x; \therefore x = 20;$$

hence B 's score was 20, and A 's 11; i.e. A was beaten by 9.

60. Let x be the number of oranges;

\therefore he pays for them $\frac{x}{2}$ pence;

he sells $x - 50$ oranges for $\frac{2}{3}(x - 50)$ pence,

$$\therefore \frac{2}{3}(x - 50) = \frac{x}{2} + 129;$$

$$4x - 200 = 3x + 774; \therefore x = 974.$$

EXERCISE XI.

1. Multiply equation (2) by 8; $8x + 9y = 51$;
 subtract equation (1); $3x + 4y = 26$;
 $\therefore 5y = 25$; $\therefore y = 5$;
 substitute this value of y in equation (2),
 then $x + 15 = 17$; $\therefore x = 2$.

2. From (1), $4x + 8y = 76$;
 subtract (2), $4x + 5y = 55$;
 $\therefore 3y = 21$; $\therefore y = 7$;
 substitute this value of y in (1),
 then $x + 14 = 19$; $\therefore x = 5$.

3. From (2), $2x + 2y = 30$;
 from (1), $2x - 5y = 9$;
 $\therefore 7y = 21$; $\therefore y = 3$;
 substitute in (2); then $x + 3 = 15$; $\therefore x = 12$.

4. From (1), $3x + 27y = 63$;
 from (2), $3x + 5y = 19$;
 $\therefore 22y = 44$; $\therefore y = 2$;
 substitute in (1); then $x + 18 = 21$; $\therefore x = 3$.

5. From (2), $8x + 2y = 96$;
 from (1), $3x + 2y = 46$;
 $\therefore 5x = 50$; $\therefore x = 10$;
 substitute in (2); then $40 + y = 48$; $\therefore y = 8$.

6. From (1), $14x - 7y = 112$;
 from (2), $8x - 7y = 35$;
 $\therefore 11x = 77$; $\therefore x = 7$;
 substitute in (1); then $14 - y = 16$; $\therefore y = -2$.

7. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 6x + 10y &= 74; \\ 6x + 9y &= 69; \\ \therefore y &= 5; \\ 8x + 25 &= 37; \\ \therefore 8x &= 12; \text{ or } x = 1.5 \end{aligned}$$

8. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 12x + 15y &= 129; \\ 12x + 8y &= 80; \\ \therefore 7y &= 49; \therefore y = 7; \\ 4x + 35 &= 43; \\ \therefore 4x &= 8, \text{ or } x = 2. \end{aligned}$$

9. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 14x + 4y &= 142; \\ 14x + 49y &= 7; \\ \therefore -45y &= 135; \therefore y = -3; \\ 7x - 6 &= 71; \therefore x = 11. \end{aligned}$$

10. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 28x - 85y &= 0; \\ 28x + 24y &= 236; \\ \therefore -59y &= -236; \therefore y = 4; \\ 4x - 20 &= 0; \therefore x = 5. \end{aligned}$$

11. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 55x - 22y &= 275; \\ 16x - 22y &= 2; \\ \therefore 39x &= 273; \therefore x = 7; \\ 85 - 2y &= 25; \therefore y = 5. \end{aligned}$$

12. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 30x - 15y &= 225; \\ 21x - 15y &= 45; \\ \therefore 9x &= 180; \therefore x = 20; \\ 120 - 3y &= 45; \therefore y = 25. \end{aligned}$$

13. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 20x + 6y &= 164; \\ 63x + 6y &= 465; \\ \therefore -43x &= -301; \therefore x = 7; \\ 70 + 3y &= 82; \therefore y = 4. \end{aligned}$$

14. From (1),
from (2),

substitute in (2); then

$$\begin{aligned} 30x + 8y &= 198; \\ 30x - 25y &= 0; \\ \therefore 33y &= 198; \therefore y = 6; \\ 6x - 30 &= 0; \therefore x = 5. \end{aligned}$$

15. From (1),
from (2),

substitute in (1); then

$$\begin{aligned} 84x + 35y &= 91; \\ 75x + 35y &= 100; \\ \therefore 9x &= -9; \therefore x = -1; \\ -12 + 5y &= 13; \therefore y = 5. \end{aligned}$$

16. From (1),
from (2),
substitute in (1); then

$$\begin{aligned}8x - 12y &= 140; \\51x - 12y &= 54; \\ \therefore -43x &= 86; \therefore x = -2; \\ -4 - 3y &= 35; \therefore y = -13.\end{aligned}$$

17. From (1),
from (2),
 \therefore by addition,
substitute in (1); then

$$\begin{aligned}8x - 12y &= 284; \\27x + 12y &= 66; \\35x &= 350; \therefore x = 10; \\20 - 3y &= 71; \therefore y = -17.\end{aligned}$$

18. From (1),
from (2),
substitute in (1); then

$$\begin{aligned}20x + 35y &= 100; \\20x - 44y &= 100; \\ \therefore 79y &= 0; \therefore y = 0; \\4x &= 20; \therefore x = 5.\end{aligned}$$

19. From (1),
from (2),
 \therefore by addition,
substitute in (1); then

$$\begin{aligned}15x - 12y &= 84; \\82x + 12y &= -84; \\47x &= 0; \therefore x = 0; \\-4y &= 28; \therefore y = -7.\end{aligned}$$

20. From (2),
from (1),
 \therefore by addition,
substitute in (1); then

$$\begin{aligned}84x - 7y &= 623, \\5x + 7y &= 0; \\89x &= 623; \therefore x = 7; \\35 + 7y &= 0; \therefore y = -5.\end{aligned}$$

21. From (2),
whence,
from (1),
 \therefore by subtraction,
substitute in (1); then

$$\begin{aligned}4x + 9y &= 3; \\12x + 27y &= 9; \\12x + 22y &= 14; \\5y &= -5, \text{ or } y = -1; \\6x - 11 &= 7; \therefore x = 3.\end{aligned}$$

22. Equation (2) may be written
whence,
from (1),
 \therefore by addition,
substitute in (2); then

$$\begin{aligned}-15x + 4y &= 137; \\-60x + 16y &= 548; \\60x + 68y &= 84; \\79y &= 632, \text{ or } y = 8; \\-15x + 32 &= 137; \therefore x = -7.\end{aligned}$$

23. The equations may be written
from (2),
from (1),
 \therefore by addition,
substitute in (2); then

$$\begin{aligned}18x - 25y &= 50, \\27x + 5y &= -10; \\135x + 25y &= -50; \\18x - 25y &= 50; \\153x &= 0, \text{ or } x = 0; \\0 + 5y &= -10; \therefore y = -2.\end{aligned}$$

24. Write the equations, $-7x + 10y = -56$,
 $15x - 6y = 26$;
 from (1), $-28x + 40y = -224$,
 from (2), $75x - 40y = 130$;
 \therefore by addition, $47x = -94$, or $x = -2$;
 substitute in (1); then $14 + 10y = -56$; $\therefore y = -7$.

25. Write the equations, $11x + 9y = 2$,
 $21x - 6y = 27$;
 from (1), $22x + 18y = 4$,
 from (2), $63x - 18y = 81$;
 \therefore by addition, $85x = 85$, or $x = 1$;
 substitute in (1); then $11 + 9y = 2$; $\therefore y = -1$.

26. From (1), $12x - 38y = 100$,
 from (2), $12x - 15y = 87$;
 \therefore by subtraction, $-13y = 13$, or $y = -1$;
 substitute in (1); then $3x + 7 = 25$, or $x = 6$.

27. Write the equations, $8x - 3y = 59$,
 $-7x + 5y = 35$;
 from (1), $40x - 15y = 295$,
 from (2), $-21x + 15y = -105$;
 \therefore by addition, $19x = 190$, or $x = 10$;
 substitute in (1), $80 - 3y = 59$; $\therefore y = 7$.

28. Write the equations, $5x + 4y = 25$,
 $3x - 7y = 15$;
 from (1), $15x + 12y = 75$,
 from (2), $15x - 35y = 75$;
 $\therefore 47y = 0$, or $y = 0$;
 substitute in (1); then $5x = 25$, or $x = 5$.

29. Write the equations, $6x + 7y = 80$,
 $4x + 11y = 66$;
 from (1), $12x + 14y = 160$,
 from (2), $12x + 33y = 198$;
 $\therefore -19y = -38$, or $y = 2$;
 substitute in (1); then $12x + 28 = 160$; $\therefore x = 11$.

30. Write the equations, $13x + 10y = 103$,
 $9x - y = 0$;
 from (2), $90x - 10y = 0$,
 from (1), $13x + 10y = 103$;
 $\therefore 103x = 103$, or $x = 1$;
 substitute in (2); then $9 - y = 0$, or $y = 9$.

31. The equations are $3x - y = 14$,
 $2x + 4y = 14$;
 from (1), $12x - 4y = 56$,
 from (2), $2x + 4y = 14$;
 $\therefore 14x = 70$, or $x = 5$;
 $10 + 4y = 14$, or $y = 1$.
 substitute in (2); then
32. The equations are $5x + 3y = 11$,
 $-2x + y = 11$;
 from (2), $-6x + 3y = 33$,
 from (1), $5x + 3y = 11$;
 $\therefore -11x = 22$, or $x = -2$;
 $4 + y = 11$, or $y = 7$.
 substitute in (2); then
33. The equations are $2x - 16 = 5y$,
 and $2x - 16 = 3x - 24$;
 from (2), $x = 8$;
 substitute in (1); then $16 - 16 = 5y$; $\therefore y = 0$.
34. The equations are $7y = 8x - 5$,
 and $8x - 5 = 10x - 1$;
 from (2), $2x = -4$, or $x = -2$;
 substitute in (1); then $7y = -16 - 5$; $\therefore y = -3$.
35. In equation (1), L. C. D. is 8,
 and the equation becomes $x + 2y = 28$;
 in equation (2), L. C. D. is 24,
 and the equation becomes $3x + 8y = 96$;
 from (1), $3x + 6y = 84$,
 from (2), $3x + 8y = 96$;
 $\therefore -2y = -12$, or $y = 6$;
 $x + 12 = 28$; $\therefore x = 16$.
 substitute in (1); then
36. The equations become $2x - y = 10$;
 $8x - 3y = 60$;
 from (1), $8x - 4y = 40$,
 from (2), $8x - 3y = 60$;
 $\therefore -y = -20$, or $y = 20$;
 $2x - 20 = 10$; $\therefore x = 15$.
 substitute in (1); then
37. The equations become $8x + 4y - 9y = 36$, or $8x - 5y = 36$,
 and $7x - 7y + 3x = 42$, or $10x - 7y = 42$;
 from (1), $40x - 25y = 180$,
 from (2), $40x - 28y = 168$;
 $\therefore 3y = 12$, or $y = 4$;
 substitute in (1); then $8x - 20 = 36$; $\therefore x = 7$.

38. The equations become $7x - 5x + 10y = 0$, or $2x + 10y = 0$;
 and $12y + 20y - 5x - 5y = 40$, or $7x + 15y = 40$;
 from (1), $6x + 30y = 0$,
 from (2), $14x + 30y = 80$;
 $\therefore -8x = -80$, or $x = 10$;
 substitute in (1); then, $20 + 10y = 0$; $\therefore y = -2$.

39. The equations become $6x + 14y - x - y = 40$, or $5x + 13y = 40$;
 and $7x + 3y = 56$;
 from (1), $35x + 91y = 280$;
 from (2), $35x + 15y = 280$;
 $\therefore 76y = 0$, or $y = 0$;
 substitute in (1); then $5x = 40$; $\therefore x = 8$.

40. Equation (1) becomes $3x - 6y - 8x + 28 = 86$, or $-5x - 6y = 8$;
 equation (2) becomes $-x + y = 17$;
 from (1), $-5x + 5y = 85$;
 from (1), $-5x - 6y = 8$;
 $\therefore 11y = 77$, or $y = 7$;
 substitute in (2); then $-x + 7 = 17$; $\therefore x = -10$.

41. Equation (1) becomes $15x - 5y - 28x + 12y = 6$; or $-13x + 7y = 6$;
 equation (2) becomes $5x - 2y = 6$;
 from (1), $-26x + 14y = 12$;
 from (2), $35x - 14y = 42$;
 $\therefore 9x = 54$, or $x = 6$;
 substitute in (2); then $30 - 2y = 6$; $\therefore y = 12$.

42. Equation (1) becomes $32x + x - y = 168$, or $33x - y = 168$;
 equation (2) becomes $6y + 3x - 5y = 12$, or $3x + y = 12$;
 \therefore by addition, $36x = 180$, or $x = 5$;
 substitute in (2); then $15 + y = 12$; $\therefore y = -3$.

43. Equation (1) becomes $5x + 8x - 4y = 50$; or $13x - 4y = 50$;
 equation (2) becomes $8y + 7x - 14y = 0$; or $7x - 6y = 0$;
 from (1), $39x - 12y = 150$;
 from (2), $14x - 12y = 0$;
 $\therefore 25x = 150$, or $x = 6$;
 substitute in (1), $78 - 4y = 50$; $\therefore y = 7$.

44. Equation (1) becomes $12x - 15x + 9y = 2y + 6$; or $-3x + 7y = 6$;
 equation (2) becomes $12y - 7x + y + 4x = 6$; or $-3x + 13y = 6$;
 \therefore by subtraction, $6y = 0$, or $y = 0$;
 substitute in (1); then $-3x = 6$, or $x = -2$.

45. Equation (1) becomes $2x - 2y - 6 = 6y - 27x$; or $29x - 8y = 6$;
 equation (2) becomes $4x - 3y = 16$;
 from (1), $87x - 24y = 18$;
 from (2), $32x - 24y = 128$;
 $\therefore 55x = -110$, or $x = -2$;
 substitute in (2); then $-8 - 3y = 16$; $\therefore y = -8$.

46. Equation (1) becomes $4x - 16y - 5x - 45y = 50$; or $-x - 61y = 50$;
 equation (2) becomes $4x + 14y = 3x + 3y$, or $x + 11y = 0$;
 \therefore by addition, $-50y = 50$, or $y = -1$;
 substitute in (2); then $x - 11 = 0$; $\therefore x = 11$.

47. Equation (1) becomes $44x - 11y = 14x + 6y + 11$; or $30x - 17y = 11$;
 equation (2) becomes $2x + 2x + 3y = 8x$; or $-4x + 3y = 0$;
 from (1), $60x - 34y = 22$,
 from (2), $-60x + 45y = 0$,
 $\therefore 11y = 22$, or $y = 2$;
 substitute in (2); then $-4x + 6 = 0$; $\therefore x = 1\frac{1}{2}$.

48. Equation (1) becomes $50x - 49x - 28y = 140$; or $x - 28y = 140$;
 equation (2) becomes $2x - 4y - 10x - 5y = 45$; $-8x - 9y = 45$;
 from (1), $8x - 224y = 1120$,
 from (2), $-8x - 9y = 45$;
 $\therefore -233y = 1165$, or $y = -5$;
 substitute in (1), then $x + 140 = 140$, or $x = 0$.

49. From (2), $\frac{5}{x} + \frac{30}{y} = 10$;
 from (1), $\frac{5}{x} + \frac{12}{y} = 7$;
 \therefore by subtraction, $\frac{18}{y} = 3$, or $y = 6$;
 substitute in (1); then $\frac{5}{x} + 2 = 7$; $\therefore x = 1$.

50. From (1), $\frac{24}{x} - \frac{4}{y} = 6$;
 from (2), $\frac{10}{x} - \frac{4}{y} = 3\frac{1}{3}$;
 \therefore by subtraction, $\frac{14}{x} = \frac{7}{3}$; or $x = 6$;
 substitute in (1); then $2 - \frac{2}{y} = 3$; $\therefore y = -2$.

51. From (1), $\frac{14}{x} - \frac{49}{y} = 35$;
 from (2), $\frac{14}{x} - \frac{24}{y} = 10$;
 $\therefore -\frac{25}{y} = 25$, or $y = -1$;
 substitute in (1); then $\frac{2}{x} + 7 = 5$; $\therefore x = -1$.

52. Equation (1) becomes $\frac{16}{x} - \frac{20}{y} = \frac{10}{8}$;
 also dividing equation (2) by xy , we have $\frac{6}{x} + \frac{5}{y} = 0$;
 from (2), $\frac{12}{x} + \frac{10}{y} = 0$,
 from (1), $\frac{8}{x} - \frac{10}{y} = \frac{5}{8}$;
 $\therefore \frac{20}{x} = \frac{5}{8}$, or $x = 12$;
 substitute in (2); then $\frac{1}{2} + \frac{5}{y} = 0$; $\therefore y = -10$.

53. Equation (1) becomes $8x + 24y - 6x + 21y = 8$; or $2x + 45y = 8$;
 equation (2) becomes $x - 3y = 4$;
 from (2), $2x - 6y = 8$,
 from (1), $2x + 45y = 8$;
 $\therefore -51y = 0$, or $y = 0$,
 substitute in (1); then $2x = 8$, or $x = 4$.

54. Equation (1) becomes $36x - 6x + 3y = 10 - 4y$, or $30x + 7y = 10$;
 equation (2) becomes (with L. C. D. 60)
 $50 - 20y = 15x - 15y - 120 - 6x - 6y + 12$, or $-9x + y = -158$;
 from (2), $-63x + 7y = -1106$,
 from (1), $30x + 7y = 10$;
 $\therefore -93x = -1116$, or $x = 12$,
 substitute in (2); then $-108 + y = -158$; $\therefore y = -50$.

EXERCISE XII.

1. Let x and y be the two numbers;
 then $x + 2y = 81$,
 and $y + 2x = 26$;
 from (1), $2x + 4y = 62$,
 from (2), $2x + y = 26$;
 $\therefore 3y = 36$, or $y = 12$
 and by substitution in (1), $x = 7$.
2. Let x and y be the two numbers;
 then $y + 2x = 17$,
 and $2y + x = 19$;
 from (1), $2y + 4x = 34$,
 from (2), $2y + x = 19$;
 $\therefore 3x = 15$, or $x = 5$;
 and by substitution in (1), $y = 7$.
3. Let x and y be the two numbers;
 then $2x - y = 30$,
 and $5y - x = 3$;
 from (2), $10y - 2x = 6$,
 from (1), $2x - y = 30$;
 \therefore by addition, $9y = 36$, or $y = 4$;
 substitute in (1), then $2x - 4 = 30$; or $x = 17$.
4. Let the price of a horse be x £s,
 and cow be y £s;
 then $9x + 7y = 300$,
 and $6x + 13y = 300$;
 from (1), $18x + 14y = 600$,
 from (2), $18x + 39y = 900$;
 $\therefore -25y = -300$, or $y = 12$;
 and by substitution, $9x + 84 = 300$, or $x = 24$.
5. Let the prices of the two sorts be x pence and y pence per lb.
 respectively;
 then $7x = 9y$,
 and $9x + 7y = 65$;
 from (1), $63x - 81y = 0$,
 from (2), $63x + 49y = 455$;
 $\therefore -130y = -455$, or $y = 3\frac{1}{2}$;
 \therefore by substitution in (1), $x = 4\frac{1}{2}$.

6. Let x and y be the two numbers;

then $\frac{x}{2} - y = 2,$

and $\frac{x}{3} - \frac{y}{2} = 2;$

from (1), $2x - 4y = 8,$

from (2), $2x - 3y = 12;$

$\therefore -y = -4, \text{ or } y = 4;$

\therefore by substitution in (1), $x = 12.$

7. Let x £s and y £s be the prices of A 's and B 's houses respectively;

then $x = y + 400,$

and $\frac{x}{2} = y - 600;$

$\therefore \frac{x}{2} = 1000, \text{ or } x = 2000;$

and by substitution in (1), $y = 1600.$

8. Let x years and y years be the respective ages;

then $y - \frac{x}{2} = 4,$

and $\frac{y}{5} = \frac{x}{8};$

by substitution in (1), $\frac{5x}{8} - \frac{x}{2} = 4, \text{ or } x = 32;$

\therefore from (2), $y = 20.$

9. Let x years and y years be the respective ages;

then $x + 31 = 3y,$

and $x + 1 = 2(y - 1);$

\therefore by subtraction, $30 = y + 2, \text{ or } y = 28;$

and from (1), $x = 84 - 31 = 53.$

10. Let x years and y years be the respective ages;

then $x - 4 = 3y,$

and $x = 2(y + 8);$

\therefore by subtraction, $-4 = y - 16, \text{ or } y = 12;$

\therefore from (1), $x = 40.$

11. Let x years and y years be the respective ages;

then $x - 6 = 12(y - 6),$

and $x + 3 = 3(y + 3);$

\therefore by subtraction, $-9 = 9y - 81, \text{ or } y = 8,$

also, from (1), $x - 6 = 24, \text{ or } x = 30.$

12. Let x and y be the numbers of persons in each class;
 then $x + y = 36$,
 and $3x + \frac{5y}{2} = 100$;
 from (1), $6x + 5y = 216$,
 from (2), $6x + 5y = 200$;
 $\therefore y = 16$; and from (1), $x = 20$.

13. Let x be the number of oxen, and y the number of sheep;
 $\therefore x + y = 35$,
 and $\frac{25x}{2} + \frac{9y}{4} = 191\frac{1}{4}$;
 from (1), $9x + 9y = 315$,
 from (2), $50x + 9y = 766$;
 $\therefore 41x = 451$; or $x = 11$;
 \therefore from (1), $y = 24$.

14. Let x miles be the distance from A to B ,
 and y miles B to C ;
 then $x - \frac{y}{5} = 12$,
 and $x + y = 24$;
 \therefore by subtraction, $\frac{6y}{5} = 12$, or $y = 10$;
 \therefore from (2), $x = 14$.

15. Let x and y be the digits, so that $10x + y$ is the number;
 then $x + y = 10$,
 and $10x + y = 30(x - y) + 4$;
 from (2), $20x - 31y = -4$,
 from (1), $20x + 20y = 200$;
 $\therefore -51y = -204$, or $y = 4$;
 \therefore from (1), $x = 6$, and the number is 64.

16. Let x and y be the digits, so that $10x + y$ is the number;
 then $x - y = 4$,
 and $4(x + y) = 10y + x$;
 from (2), $3x - 6y = 0$, or $x = 2y$;
 \therefore by substitution in (1), $y = 4$, and $x = 8$;
 hence 84 is the required number.

17. Let x and y be the digits, so that $10x+y$ is the number;
 then $10x+y=7(x+y)$,
 also $10y+x=10(x-y)+6$;
 from (1), $8x-6y=0$, or $9x-18y=0$;
 from (2), $-9x+20y=6$;
 \therefore by addition $2y=6$, or $y=3$;
 \therefore by substitution in (1), $x=6$; hence the number is 63.

18. Let x be the first digit, and y the middle digit;
 then $x+y+x=7$,
 and $100x+10y+x+90=100y+10x+x$;
 from (1), $2x+y=7$,
 from (2), $90x-90y=-90$, or $x-y=-1$;
 $\therefore 3x=6$, or $x=2$;
 also from (1), $y=3$; hence the number is 232.

19. Let x £s and y £s be the prices of a sheep and a calf respectively;
 then $5x+8y=80$,
 and $4\left(x+\frac{1}{2}\right)+6(y+1)=81$;
 from (2), $4x+6y=81-6-2=23$;
 or $16x+24y=92$,
 also from (1), $15x+24y=90$;
 $\therefore x=2$,
 and from (1), $8y=20$, or $y=2\frac{1}{2}$.

20. Let x feet be the length of the rectangle, and let y feet be its width;
 then $2x+2y=100$,
 and $(x+2)(y-2)=xy-24$;
 from (2), $-2x+2y=-24+4=-20$,
 from (1), $2x+2y=100$;
 \therefore by subtraction $4x=120$, or $x=30$.

21. Let x and y be the numbers of the two sorts;
 then $x+y=400$,
 also $\frac{2}{3}x+\frac{3}{2}y=x+y$
 (because each of these represents the *value* of the oranges in pence);
 now from (2), $-2x+3y=0$,
 and from (1), $2x+2y=800$;
 $\therefore 5y=800$, or $y=160$;
 \therefore from (1), $x=240$.

22. Let x £s be the income,
and y £s the expenditure;

then $y + 50 = \frac{3}{4}x$,

and $x + 225 = 2(y - 15)$;

from (1), $3x - 4y = 200$;

from (2), $x - 2y = -255$; or $2x - 4y = -510$;

hence by subtraction, $x = 710$.

23. Let x and y be the crews of the two ships;

then $\frac{x}{5} + \frac{y}{3} = \frac{2x}{3}$,

also $\frac{2x}{3} = x - 10$;

from (2), $\frac{x}{3} = 10$, or $x = 30$,

from (1), $3x + 5y = 10x$, or $5y = 7x$; $\therefore y = 42$.

24. Let x be the number of half-guineas, and y the number of half-crowns; then, expressing their value in shillings, we have

$$\frac{21x}{2} + \frac{5y}{2} = 112,$$

also $\frac{21}{2}(x+3) + \frac{15y}{2} = 178\frac{1}{2}$;

from (1), $21x + 5y = 224$,

from (2), $21x + 15y = 357 - 63 = 294$;

$$\therefore -10y = -70, \text{ or } y = 7;$$

also, by substituting in (1), $21x = 189$, or $x = 9$.

25. Let x yards be the length dug in one day,
and y yards filled up in one night;
then, since 945 yds. are finished in 21 days, 45 yds. are finished in one day;

$$\therefore x - y = 45,$$

also $2x - y = 105$;

\therefore by subtraction, $x = 60$.

26. Let x days be required by B ,
and y days by C ;
then, in one day B and C together do $\left(\frac{1}{x} + \frac{1}{y}\right)$ of the work,

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{4};$$

also in one day, A and C together do twice the amount which B does,

$$\therefore \frac{1}{12} + \frac{1}{y} = \frac{2}{x};$$

hence by subtraction, $\frac{1}{x} - \frac{1}{12} = \frac{1}{4} - \frac{2}{x}$, or $\frac{3}{x} = \frac{1}{3}$; $\therefore x = 9$;

and by substitution in (1), $\frac{1}{y} = \frac{1}{4} - \frac{1}{9} = \frac{5}{36}$;
 $\therefore y = 7\frac{1}{5}$.

27. Let x £ be what each has at first,
and y £ be what A wins;
then $x + y + 6 = 3(x - y)$,
and $x + y - 16 = 2(x - y)$;
from (1), $2x - 4y = 6$,
from (2), $x - 3y = -16$; or $2x - 6y = -32$;
 \therefore by subtraction, $2y = 38$, or $y = 19$;
 $\therefore x = 57 - 16 = 41$.

28. Let x be the number of strawberries,
and y pence the price of a pint;
then $\frac{x+6}{4} = y$,
also $y + 1 = \frac{x}{3}$;
from (1), $x = 4y - 6$,
from (2), $x = 3y + 3$;
 $\therefore y = 9$, and $x = 30$.

29. Let x be the number of volumes,
and y shillings the cost of each volume;
then $\frac{2}{3}xy + 2000 = x(y + 1)$,

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$$\begin{aligned} \text{also} \quad & \frac{3}{4}xy + 2000 = x(y+2); \\ \text{from (1),} \quad & xy + 3x = 6000, \\ \text{from (2),} \quad & xy + 8x = 8000; \\ & \therefore 5x = 2000, \text{ or } x = 400; \\ \text{and from (1),} \quad & 400y = 4800, \text{ or } y = 12x. \end{aligned}$$

$$\begin{aligned} 80. \quad & \text{Let } x \text{ be the number of divisions,} \\ & \text{and } y \text{ the average number of boys in a division;} \\ \text{then} \quad & xy + 86 = x(y+8), \\ \text{and} \quad & \frac{xy}{x-8} = 2(x-8); \\ \text{from (1),} \quad & 3x = 86, \text{ or } x = 12, \\ \therefore \text{ from (2),} \quad & \frac{xy}{9} = 18; \therefore xy = 162. \end{aligned}$$

EXERCISE XIII.

$$\begin{aligned} 1. \quad & \begin{array}{r} x^3 + 4x + 3 \mid x^3 + 5x + 4 \quad (1 \\ \underline{x^3 + 4x + 3} \\ x + 1 \end{array} \\ & \begin{array}{r} x + 1 \mid x^3 + 4x + 3 \quad (x + 3 \\ \underline{x^3 + x} \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array} \\ & x + 1 \text{ is G. C. M.} \\ 2. \quad & \begin{array}{r} x^3 + 2x + 1 \mid 2x^3 + 5x + 3 \quad (2 \\ \underline{2x^3 + 4x + 2} \\ x + 1 \end{array} \\ & \begin{array}{r} x + 1 \mid x^3 + 2x + 1 \quad (x + 1 \\ \underline{x^3 + x} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array} \\ & \text{G. C. M. } x + 1. \\ 3. \quad & \begin{array}{r} 3x^3 + 10x + 8 \mid 6x^3 + 23x + 20 \quad (2 \\ \underline{6x^3 + 20x + 16} \\ 3x + 4 \end{array} \\ & \begin{array}{r} 3x + 4 \mid 3x^3 + 10x + 8 \quad (x + 2 \\ \underline{3x^3 + 4x} \\ 6x + 8 \\ \underline{6x + 8} \\ 0 \end{array} \\ & \text{G. C. M. } 3x + 4. \end{aligned}$$

4.
$$\begin{array}{r} 5x^2 + 7x + 2 \overline{) 15x^2 + 26x + 8} \\ \underline{15x^2 + 21x + 6} \\ 5x + 2 \overline{) 5x^2 + 7x + 2} \\ \underline{5x^2 + 2x} \\ 5x + 2 \end{array}$$

G. C. M. $5x + 2$.
5.
$$\begin{array}{r} 9x^2y^2 + 18xy + 5 \overline{) 18x^2y^2 + 36xy + 15} \\ \underline{18x^2y^2 + 36xy + 10} \\ 3xy + 5 \overline{) 9x^2y^2 + 18xy + 5} \\ \underline{9x^2y^2 + 15xy} \\ 3xy + 5 \overline{) 3xy + 5} \\ \underline{3xy + 5} \end{array}$$

G. C. M. $3xy + 5$.
6.
$$\begin{array}{r} a^2 - 11a + 10 \overline{) a^2 - 10a + 9} \\ \underline{a^2 - 11a + 10} \\ a - 1 \overline{) a^2 - 11a + 10} \\ \underline{a^2 - a} \\ -10a + 10 \overline{) -10a + 10} \\ \underline{-10a + 10} \end{array}$$

G. C. M. $a - 1$.
7.
$$\begin{array}{r} 3a^2b^2 - 5ab + 2 \overline{) 3a^2b^2 - 4ab + 1} \\ \underline{3a^2b^2 - 5ab + 2} \\ ab - 1 \overline{) 3a^2b^2 - 5ab + 2} \\ \underline{3a^2b^2 - 3ab} \\ -2ab + 2 \overline{) -2ab + 2} \\ \underline{-2ab + 2} \end{array}$$

G. C. M. $ab - 1$.
8.
$$\begin{array}{r} 10x^2 - 23x + 12 \overline{) 30x^2 - 67x + 33} \\ \underline{30x^2 - 69x + 36} \\ 2x - 3 \overline{) 10x^2 - 23x + 12} \\ \underline{10x^2 - 15x} \\ -8x + 12 \overline{) -8x + 12} \\ \underline{-8x + 12} \end{array}$$

G. C. M. $2x - 3$.
9.
$$\begin{array}{r} 7x^2 + 16x - 15 \overline{) 28x^2 + 71x - 65} \\ \underline{28x^2 + 64x - 60} \\ 7x - 5 \overline{) 7x^2 + 16x - 15} \\ \underline{7x^2 - 5x} \\ 21x - 15 \overline{) 21x - 15} \\ \underline{21x - 15} \end{array}$$

G. C. M. $7x - 5$.
10.
$$\begin{array}{r} 12x^2y^2 - 19xy - 21 \overline{) 48x^2y^2 - 78xy - 91} \\ \underline{48x^2y^2 - 76xy - 84} \\ 3xy - 7 \overline{) 12x^2y^2 - 19xy - 21} \\ \underline{12x^2y^2 - 28xy} \\ 9xy - 21 \overline{) 9xy - 21} \\ \underline{9xy - 21} \end{array}$$

G. C. M. $3xy - 7$.

11.
$$\begin{array}{r} 3x^3 + 5x - 8 \overline{) 6x^3 + 19x + 8} \quad (2 \\ \underline{6x^3 + 10x - 16} \\ 8 \overline{) 9x + 24} \\ 8x + 8 \overline{) 8x^3 + 5x - 8} \quad (x - 1 \\ \underline{8x^3 + 8x} \\ - 8x - 8 \\ - 8x - 8 \end{array}$$

G. C. M. $8x + 8$.
12.
$$\begin{array}{r} 2x^3 - 13x - 7 \overline{) 2x^3 - x^2 + 3x + 2} \quad (x + 6 \\ \underline{2x^3 - 13x^2 - 7x} \\ 12x^2 + 10x + 2 \\ \underline{12x^2 - 78x - 42} \\ 44 \overline{) 88x + 44} \\ 2x + 1 \overline{) 2x^3 - 13x - 7} \quad (x - 7 \\ \underline{2x^3 + x} \\ - 14x - 7 \\ - 14x - 7 \end{array}$$

G. C. M. $2x + 1$.
13.
$$\begin{array}{r} 11x^3 + 87xy - 8y^3 \overline{) 22x^3 - 13x^2y - 10xy^2 + y^3} \quad (2x - 17y \\ \underline{22x^3 + 174x^2y - 16xy^3} \\ - 187x^2y + 6xy^3 + y^3 \\ \underline{- 187x^2y - 1479xy^2 + 136y^3} \\ 135y^3 \overline{) 1485xy^2 - 135y^3} \\ 11x - y \overline{) 11x^3 + 87xy - 8y^3} \quad (x + 8y \\ \underline{11x^3 - xy} \\ 88xy - 8y^3 \\ \underline{88xy - 8y^3} \end{array}$$

G. C. M. $11x - y$.
14.
$$\begin{array}{r} 12x^3 + 35x + 22 \overline{) 24x^3 - 14x - 33} \quad (2 \\ \underline{24x^3 + 70x + 44} \\ - 7 \overline{) - 84x - 77} \\ 12x + 11 \overline{) 12x^3 + 35x + 22} \quad (x + 2 \\ \underline{12x^3 + 11x} \\ 24x + 22 \\ \underline{24x + 22} \end{array}$$

G. C. M. $12x + 11$.
15.
$$\begin{array}{r} 2a^3 + 19a + 35 \overline{) 6a^3 + 17a + 5} \quad (3 \\ \underline{6a^3 + 57a + 105} \\ - 20 \overline{) - 40a - 100} \\ 2a + 5 \overline{) 2a^3 + 19a + 35} \quad (a + 7 \\ \underline{2a^3 + 5a} \\ 14a + 35 \\ \underline{14a + 35} \end{array}$$

G. C. M. $2a + 5$.

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$$\begin{array}{r} 16. \quad 8x^2 + 18ax - 5a^2 \overline{) 24x^3 - 10ax + a^2} \begin{array}{l} 3 \\ 24x^3 + 54ax - 15a^2 \\ \hline -16a \end{array} \overline{) -64ax + 16a^2} \begin{array}{l} 4x - a \\ 4x - a \\ \hline 8x^2 + 18ax - 5a^2 \end{array} \overline{) 2x + 5a} \begin{array}{l} 2x + 5a \\ 2x + 5a \\ \hline 0 \end{array} \\ \text{G. C. M. } 4x - a. \end{array}$$

$$\begin{array}{r}
 17. \quad \begin{array}{r} 10x^3 - 37x - 36 \end{array} \overline{) 20x^3 - 19x - 28} \begin{array}{r} 2 \\ 20x^3 - 74x - 72 \end{array} \\
 \underline{ 11} \begin{array}{r} 55x + 44 \\ 5x + 4 \end{array} \overline{) 10x^3 - 37x - 36} \begin{array}{r} 2x - 9 \\ 10x^3 + 8x \\ - 45x - 36 \\ - 45x - 36 \end{array} \\
 \text{G. C. M. } 5x + 4.
 \end{array}$$

$$\begin{array}{r}
 18. \quad 6x^3 - 5x^2 - 2x + 1 \overline{) 12x^3 - 16x^2 - x + 5} \quad (2 \\
 \underline{12x^3 - 10x^2 - 4x + 2} \\
 -3) \overline{-6x^2 + 3x + 3} \\
 \underline{2x^2 - x - 1} \quad (6x^2 - 5x^2 - 2x + 1) 3x - 1 \\
 \underline{6x^2 - 3x^2 - 3x} \\
 \underline{-2x^2 + x + 1} \\
 \text{G. C. M. } 2x^2 - x - 1.
 \end{array}$$

19.
$$\frac{2x^3 - 7x^2 - 29x - 6}{6x^3 - 21x^2 - 87x - 18} = \frac{17}{17} \frac{34x^3 + 85x + 17}{2x^3 + 5x + 1} = \frac{2x^3 - 7x^2 - 29x - 6}{2x^3 + 5x^2 + x} \cdot \frac{x}{x} = \frac{2x^3 - 7x^2 - 29x - 6}{-12x^2 - 30x - 6} = \frac{2x^3 - 7x^2 - 29x - 6}{-12x^2 - 30x - 6}$$

20. Divide one expression by 7, and the other by x :

$$\begin{array}{r} x^3 - 6x^2a + 11xa^2 - 12a^3 \quad 3x^3 - 13x^2a + 23xa^2 - 21a^3 \quad (3 \\ \underline{3x^3 - 18x^2a + 33xa^2 - 36a^3} \\ 5a) 5x^2a - 10xa^2 + 16a^3 \\ \underline{x^2 - 2xa + 3a^2} \quad x^2 - 6x^2a + 11xa^2 - 12a^2(x - 4a \\ \underline{x^2 - 2x^2a + 3xa^2} \\ \underline{-4x^2a + 8xa^2 - 12a^2} \\ -4x^2a + 8xa^2 - 12a^2 \end{array}$$

G. C. M. $x^2 - 2xa + 3a^2$.

$$\begin{array}{r}
 21. \quad 2x^3 - 11x^2 + 5x + 25) \quad 7x^3 - 25x^2 - 23x + 20 \\
 \underline{ 2} \\
 14x^3 - 50x^2 - 46x + 40 \quad (7 \\
 \underline{14x^3 - 77x^2 + 35x + 175} \\
 27) 27x^2 - 81x - 135 \\
 \underline{x^2 - 3x - 5} \quad 2x^3 - 11x^2 + 5x + 25(2x - 5 \\
 \underline{2x^3 - 6x^2 - 10x} \\
 - 5x^2 + 15x + 25 \\
 \underline{- 5x^2 + 15x + 25} \\
 \text{г. г. м. } x^2 - 3x - 5.
 \end{array}$$

$$\begin{array}{r}
 22. \quad 14x^3 - 19x^2 - 10x - 1 \\
 \underline{4x^4 - 12x^3 + 9x^2 - 7} \\
 28x^4 - 84x^3 + 63x^2 - 7 \quad (2x \\
 \underline{28x^4 - 38x^3 - 20x^2 - 2x} \\
 -46x^3 + 83x^2 + 2x - 7 \\
 \underline{ 7} \\
 -322x^3 + 581x^2 + 14x - 49 \quad (-23 \\
 \underline{-322x^3 + 487x^2 + 230x + 23} \\
 72) 144x^2 - 216x - 72 \\
 \underline{2x^2 - 3x - 1} \quad 14x^3 - 19x^2 - 10x - 1 \quad (7x + 1 \\
 \underline{14x^3 - 21x^2 - 7x} \\
 2x^2 - 3x - 1 \\
 \underline{2x^2 - 3x - 1} \\
 0
 \end{array}$$

[illegible]

$$24. \quad \frac{3x^3 + 8x^2 - 8x + 11}{9x^3 + 24x^2 - 24x + 33} \cdot \frac{9x^2 + 3x - 110}{9x^3 + 8x^2 - 8x + 11}$$

$$\frac{9x^3 + 24x^2 - 24x + 33}{9x^3 + 8x^2 - 110x}$$

$$\frac{21x^2 + 86x + 33}{3}$$

$$\frac{63x^3 + 258x^2 + 99}{63x^3 + 21x - 770}$$

$$\frac{79) 237x + 869}{3x + 11}$$

$$\frac{9x^2 + 3x - 110}{9x^2 + 83x}$$

$$\frac{-80x - 110}{-80x - 110}$$

G. C. M. $3x + 11$.

$$25. \quad \frac{7x^3 - 22x^2 + 31x - 4}{21x^3 - 66x^2 + 93x - 12} \cdot \frac{21x^3 - 8x^2 - 7x + 1}{63x^3 - 100x^2 + 13}$$

$$\frac{7x^3 - 22x^2 + 31x - 4}{63x^3 - 100x^2 + 13}$$

$$\frac{279x^2 - 36}{13x}$$

$$\frac{63x^3 - 198x^2 + 279x - 36}{63x^3 - 100x^2 + 13x}$$

$$\frac{-98x^2 + 266x - 36}{9}$$

$$\frac{-882x^2 + 2394x - 324}{-882x^2 + 1400x - 182}$$

$$\frac{142) 994x - 142}{7x - 1}$$

$$\frac{63x^3 - 100x^2 + 13}{63x^3 - 9x}$$

$$\frac{-91x + 13}{-91x + 13}$$

G. C. M. $7x - 1$.

$$\begin{array}{r}
 2x^3 - 3x^2 - 19x - 86 \quad | \quad 6x^3 - 41x^2 + 67x - 18(3) \\
 \underline{6x^3 - 9x^2 - 57x - 108} \\
 -2) \quad 32x^2 + 124x + 90 \\
 \underline{16x^2 - 62x - 45} \quad 2x^2 - 3x^2 - 19x - 86 \quad 8
 \end{array}$$

$$\begin{array}{r}
 16x^3 - 24x^2 - 152x - 288(x) \\
 \underline{16x^3 - 62x^2 - 45x} \\
 38x^2 - 107x - 288 \quad 8
 \end{array}$$

$$\begin{array}{r}
 304x^2 - 856x - 2304(19) \\
 \underline{304x^2 - 1178x - 855} \\
 161322x - 1449
 \end{array}$$

$$\begin{array}{r}
 2x - 9 \quad | \quad 16x^3 - 62x - 45(8x + 5) \\
 \underline{16x^3 - 72x} \\
 10x - 45 \\
 \underline{10x - 45}
 \end{array}$$

G. C. M. $2x - 9$.

27. Divide the *last* terms into each other; thus

$$\begin{array}{r}
 6x^3 - 29x^2 + 31x - 4 \quad | \quad 9x^3 - 12x^2 - 15x + 20(-5) \\
 \underline{-30x^3 + 145x^2 - 155x + 20} \\
 x) \quad 39x^3 - 157x^2 + 140x \\
 \underline{39x^3 - 157x^2 + 140x} \quad 6x^3 - 29x^2 + 31x - 4 \quad 13
 \end{array}$$

$$\begin{array}{r}
 78x^3 - 377x^2 + 403x - 52(2x) \\
 \underline{78x^3 - 314x^2 + 280x} \\
 -63x^2 + 123x - 52 \quad 13
 \end{array}$$

$$\begin{array}{r}
 -819x^3 + 1599x^2 - 676(-21) \\
 \underline{-819x^3 + 3297x^2 - 2940} \\
 -566) \quad -1698x + 2264 \\
 \underline{3x - 4}
 \end{array}$$

$$\begin{array}{r}
 39x^3 - 157x + 140(13x - 35) \\
 \underline{39x^3 - 52x} \\
 -105x + 140
 \end{array}$$

$$23. \quad \frac{3x^2 - x^2 - x - 4}{6x^3 - 2x^2 - 2x - 8} \frac{6x^3 + 13x^2 - 81x + 4}{15x^2 - 29x + 12} \frac{4}{8} \frac{2}{2} \frac{2}{2} \frac{2}{2}$$

$$\frac{15x^3 - 5x^2 - 5x - 20}{15x^3 - 29x^2 + 12x} \frac{20}{24x^2 - 17x - 20} \frac{x}{5}$$

$$\frac{120x^2 - 85x - 100}{120x^2 - 282x + 96} \frac{8}{49} \frac{147x - 196}{3x - 4}$$

$$\frac{15x^2 - 29x + 12}{15x^3 - 20x} \frac{5x - 3}{9x + 12} \frac{9x + 12}{9x + 12}$$

G. C. H. 3x - 4.

$$20. \quad \frac{22x^3 - 15x^2 - 9x + 2}{33x^3 - 61x^2 + 54x - 8} \frac{8}{2}$$

$$\frac{66x^3 - 122x^2 + 108x - 16}{66x^3 - 45x^2 - 27x + 6} \frac{16}{-77x^2 + 135x - 22} \frac{3}{6} \frac{6}{6} \frac{6}{6}$$

$$\frac{15x^2 - 9x + 2}{7}$$

$$\frac{154x^3 - 105x^2 - 63x + 14}{154x^3 - 270x^2 + 44x} \frac{-2x}{165x^2 - 107x + 14} \frac{7}{7}$$

$$\frac{1155x^2 - 749x + 98}{1155x^2 - 2025x + 330} \frac{-15}{116} \frac{1276x - 232}{11x - 2}$$

$$\frac{77x^2 - 135x + 22}{77x^2 - 14x} \frac{7x - 11}{-121x + 22} \frac{-121x + 22}{-121x + 22}$$

80. $2x^3 - 15x^2a + 9xa^2 - a^3 \mid 14x^3 - 7x^2a - 4xa^2 + 2a^3 (7$
 $14x^3 - 105x^2a + 63xa^2 - 7a^3$
 $a \mid 98x^2a - 67xa^2 + 9a^3$
 $98x^3 - 67xa + 9a^2 \mid 2x^3 - 15x^2a + 9xa^2 - a^3$
 $9 \mid 18x^3 - 135x^2a + 81xa^2 - 9a^3 (-a$
 $x \mid 18x^3 - 37x^2a + 14xa^2$
 $18x^3 - 37xa + 14a^2$
 $9 \mid 162x^3 - 333xa + 126a^2 (14$
 $1372x^3 - 938xa + 126a^2$
 $- 603x \mid - 1210x^3 + 605xa$
 $2x - a \mid 98x^3 - 67xa + 9a^2 (49x - 9a$
 $98x^3 - 49xa$
 $- 18xa + 9a^2$
 $- 18xa + 9a^2$
31. $6x^3 - 41x + 63 \mid 12x^3 - 84x^2 + 17x - 7 (2x + 8$
 $12x^3 - 82x^2 + 126x$
 $48x^3 - 109x - 7$
 $48x^3 - 328x + 504$
 $73 \mid 219x - 511$
 $8x - 7 \mid 6x^3 - 41x + 63 (2x - 9$
 $6x^3 - 14x$
 $- 27x + 63$
 $- 27x + 63$
- G. C. M. $2x - a$.
- G. C. M. $3x - 7$.

32.

Divide the first expression by x^2 ;

$$\begin{array}{r} 12x^3 - 31x + 14 \quad | \quad 86x^3 - 45x^2 + 74x - 35(3x + 4) \\ \underline{86x^3 - 93x^2 + 42x} \\ 48x^2 + 32x - 35 \\ \underline{48x^2 - 124x + 56} \\ 18 \quad | \quad 156x - 91 \end{array}$$

$$\begin{array}{r} 12x - 7 \quad | \quad 12x^3 - 31x + 14(x - 2) \\ \underline{12x^3 - 7x} \\ -24x + 14 \\ \underline{-24x + 14} \end{array}$$

G. C. M. $12x - 7$.

33.

$$\begin{array}{r} 5x^6 - x^4 + 11x^3 + 12 \quad | \quad 10x^6 - 7x^4 - 2x^2 + 8(2) \\ \underline{10x^6 - 2x^4 + 22x^3 + 24} \\ -5x^4 - 24x^3 - 16 \end{array}$$

$$\begin{array}{r} 5x^6 - 24x^3 - 16 \quad | \quad 5x^6 - x^4 + 11x^3 + 12(-x^2 + 5) \\ \underline{5x^6 + 24x^4 + 16x^3} \\ -25x^4 - 5x^2 + 12 \\ \underline{-25x^4 - 120x^2 - 80} \\ 28 \quad | \quad 115x^2 + 92 \end{array}$$

$$\begin{array}{r} 5x^2 + 4 \quad | \quad 5x^4 + 24x^3 + 16(x^2 + 4) \\ \underline{5x^4 + 4x^3} \\ 20x^3 + 16 \\ \underline{20x^3 + 16} \end{array}$$

G. C. M. $5x^2 + 4$.

84.

$$9x^3 + 2x - 1) 12x^3 + 2x^3 - 11x + 8$$

$$\frac{36x^3 + 6x^3 - 33x + 9}{86x^3 + 8x - 4} \quad \frac{9(4}{6x^3 - 41x + 13) \quad 9x^3 + 2x - 4$$

$$\frac{36x^3 + 8x - 4}{86x^3 - 246x^3 + 78x} \quad \frac{8x - 4}{6x + 41}$$

$$\frac{246x^3 - 70x - 4}{246x^3 - 1681x + 533}$$

$$\frac{537) 1611x - 537}{3x - 1) 6x^2 - 41x + 13(2x - 13}$$

$$\frac{6x^2 - 2x}{-39x + 13}$$

$$\frac{-39x + 13}{-39x + 13}$$

G. C. M. $3x - 1$.

85. Divide the second expression by y :

$$2x^3 - 11x^2y + 20xy^2 - 21y^3) \quad 9x^3 - 30x^2y + 51xy^2 - 36y^3$$

$$\frac{18x^3 - 60x^2y + 102xy^2 - 72y^3}{18x^3 - 99x^2y + 180xy^2 - 189y^3} \quad \frac{72y^3}{2} (9$$

$$\frac{89y) 39x^2y - 78xy^2 + 117y^3}{x^3 - 2xy + 3y^3) \quad 2x^3 - 11x^2y + 20xy^2 - 21y^3(2x - 7y}$$

$$\frac{2x^3 - 4x^2y + 6xy^3}{-7x^2y + 14xy^2 - 21y^3}$$

$$\frac{-7x^2y + 14xy^2 - 21y^3}{-7x^2y + 14xy^2 - 21y^3}$$

G. C. M. $x^3 - 2xy + 3y^3$.

o

D. A. K.

36. Divide the first expression by x , and the second by $6xy$; then

$$\begin{array}{r}
 8x^3 - 22x^2 - 28x + 77 \quad 6x^3 - 2x^2 - 21x + \frac{7}{4} \\
 \hline
 24x^3 - 8x^2 - 84x + 28(3) \\
 24x^3 - 66x^2 - 84x + 231 \\
 \hline
 29 \quad 58x^2 \quad - 203 \\
 \hline
 2x^2 - 7 \quad 8x^3 - 22x^2 - 28x + 77(4x - 11) \\
 \hline
 8x^3 \quad - 28x \\
 \hline
 - 22x^2 \quad + 77 \\
 - 22x^2 \quad + 77 \\
 \hline
 \end{array}$$

$\therefore 2x^2 - 7$ is a common measure, and since x is common to the two factors which were divided out at first, \therefore the g. c. m. is $x(2x^2 - 7)$.

$$\begin{array}{r}
 37. \quad x^3 + 2x^2 - x - 2 \quad x^4 + 2x^3 - 3x^2 - 4x + 4(x) \\
 \hline
 x^4 + 2x^3 - x^2 - 2x \\
 \hline
 - 2 \quad - 2x^2 - 2x + 4 \\
 \hline
 x^2 + x - 2 \quad x^3 + 2x^2 - x - 2(x + 1) \\
 \hline
 x^3 + x^2 - 2x \\
 \hline
 x^2 + x - 2 \\
 x^2 + x - 2 \\
 \hline
 \end{array}$$

$\therefore x^2 + x - 2$ is a common measure, and since 2 is g. c. m. of 6 and 8, $\therefore 2(x^2 + x - 2)$ is g. c. m.

$$\begin{array}{r}
 38. \quad x^3 - 6x^2 - 26x - 9 \quad x^4 - 2x^3 - 7x^2 + 16x + 7(x + 4) \\
 \hline
 x^4 - 6x^3 - 26x^2 - 9x \\
 \hline
 4x^3 + 19x^2 + 25x + 7 \\
 4x^3 - 24x^2 - 104x - 36 \\
 \hline
 43 \quad 43x^2 + 129x + 43 \\
 \hline
 x^2 + 3x + 1 \quad x^3 - 6x^2 - 26x - 9(x - 9) \\
 \hline
 x^3 + 3x^2 + x \\
 \hline
 - 9x^2 - 27x - 9 \\
 - 9x^2 - 27x - 9 \\
 \hline
 \end{array}$$

Also $2x$ is the g. c. m. of $2x^2$ and $6x$; $\therefore 2x(x^2 + 3x + 1)$ is the g. c. m. of the two expressions.

40. Divide the first expression by
- x
- , and the second by
- $2x^2$
- ; then

$$\begin{array}{r}
 2x^3 - 5x^2 - 7x - 2 \quad x^4 - 6x^3 + 9x^2 - \frac{4}{2} \\
 \hline
 2x^4 - 12x^3 + 18x^2 - 8 \quad (x \\
 2x^4 - 5x^3 - 7x^2 - 2x \\
 \hline
 - 7x^3 + 25x^2 + 2x - 8 \\
 \hline
 - 14x^3 + 50x^2 + 4x - 16 \quad (-7 \\
 - 14x^3 + 35x^2 + 49x + 14 \\
 \hline
 15 \quad 15x^2 - 45x - 30 \\
 \hline
 x^2 - 3x - 2 \quad 2x^3 - 5x^2 - 7x - 2 \quad (2x + 1 \\
 \hline
 2x^3 - 6x^2 - 4x \\
 \hline
 x^2 - 3x - 2 \\
 \hline
 x^2 - 3x - 2
 \end{array}$$

$\therefore x^2 - 3x - 2$ is a common measure, and x is common to x and $2x^2$,
 $\therefore x(x^2 - 3x - 2)$ is G. C. M.

EXERCISE XIV.

1. Divide numerator and denominator by $6a^4b^3$.
2. $3a^2b^4c^3$.
3. $7xy^3z^5$.
4. $5a^3b$.
5. $27ax^3$.
6. $15ax^2$.
7. $12ax^2y$.
8. $144y$.
9. $10p^2x^3$.
10. $2ab^2k$.
11. $x^2 - 4x + 3$

$$\begin{array}{r}
 x^2 - 4x + 3 \quad x^2 - 2x - 3 \quad (1 \\
 x^2 - 4x + 3 \\
 \hline
 2 \quad 2x - 6 \\
 \hline
 x - 3 \quad x^2 - 4x + 3 \quad (x - 1 \\
 x^2 - 3x \\
 \hline
 - x + 3 \\
 \hline
 - x + 3
 \end{array}$$

The G. C. M. is $x - 3$, and dividing numerator and denominator by it we obtain $\frac{x-1}{x+1}$.

$$\begin{array}{r}
 12. \quad \frac{x^2 + xy - 6y^2}{4y} \frac{x^2 + 5xy + 6y^2}{4xy + 12y^2} (1 \\
 \quad \quad \quad \frac{x^2 + xy - 6y^2}{x + 3y} \frac{x^2 + xy - 6y^2}{x^2 + 3xy} (x - 2y \\
 \quad \quad \quad \quad \quad \quad \frac{-2xy - 6y^2}{-2xy - 6y^2}
 \end{array}$$

g. c. m. is $x + 3y$, and dividing numerator and denominator we obtain

$$\frac{x + 2y}{x - 2y}.$$

$$\begin{array}{r}
 13. \quad \frac{x^3 - 8xy + 15y^2}{11y} \frac{x^2 + 3xy - 40y^2}{11xy - 55y^2} (1 \\
 \quad \quad \quad \frac{x^3 - 8xy + 15y^2}{x - 5y} \frac{x^2 + 3xy - 40y^2}{x^2 - 5xy} (x - 3y \\
 \quad \quad \quad \quad \quad \quad \frac{-3xy + 15y^2}{-3xy + 15y^2}
 \end{array}$$

Divide numerator and denominator by $x - 5y$, and we obtain $\frac{x - 3y}{x + 8y}$.

$$\begin{array}{r}
 14. \quad \frac{6a^2 - 7ab - 3b^2}{6b} \frac{6a^2 + 11ab + 3b^2}{18ab + 6b^2} (1 \\
 \quad \quad \quad \frac{6a^2 - 7ab - 3b^2}{3a + b} \frac{6a^2 + 11ab + 3b^2}{6a^2 + 2ab} (2a - 3b \\
 \quad \quad \quad \quad \quad \quad \frac{-9ab - 3b^2}{-9ab - 3b^2}
 \end{array}$$

Divide numerator and denominator by $3a + b$, and we obtain $\frac{2a - 3b}{2a + 3b}$.

$$\begin{array}{r}
 15. \quad \frac{x^2 - 6x + 8}{4x^2 - 24x + 32} \frac{4x^2 - 23x + 28}{x - 4} (4 \\
 \quad \quad \quad \frac{x^2 - 6x + 8}{x^2 - 4x} (x - 2 \\
 \quad \quad \quad \quad \quad \quad \frac{-2x + 8}{-2x + 8}
 \end{array}$$

Dividing numerator and denominator by $x - 4$, we obtain $\frac{x - 2}{4x - 7}$.

$$\begin{array}{r}
 16. \quad x^2 + 4x - 77 \overline{) x^2 + 18x + 77} \quad (1 \\
 \underline{x^2 + 4x - 77} \\
 14 \overline{) 14x + 154} \\
 \underline{x + 11} \quad x^2 + 4x - 77(x - 7) \\
 \underline{x^2 + 11x} \\
 - 7x - 77 \\
 - 7x - 77
 \end{array}$$

Dividing numerator and denominator by $x + 11$, we obtain $\frac{x-7}{x+7}$.

$$\begin{array}{r}
 17. \quad 5x^2 - 17x + 14 \overline{) 5x^2 - 7x - 6} \quad (1 \\
 \underline{5x^2 - 17x + 14} \\
 10 \overline{) 10x - 20} \\
 \underline{x - 2} \quad 5x^2 - 17x + 14(5x - 7) \\
 \underline{5x^2 - 10x} \\
 - 7x + 14 \\
 - 7x + 14
 \end{array}$$

Dividing numerator and denominator by $x - 2$, we obtain $\frac{5x+8}{5x-7}$.

$$\begin{array}{r}
 18. \quad p^2 - 7p + 12 \overline{) p^2 - 5p + 4} \quad (1 \\
 \underline{p^2 - 7p + 12} \\
 2 \overline{) 2p - 8} \\
 \underline{p - 4} \quad p^2 - 7p + 12(p - 3) \\
 \underline{p^2 - 4p} \\
 - 3p + 12 \\
 - 3p + 12
 \end{array}$$

Dividing numerator and denominator by $p - 4$, we obtain $\frac{p-1}{p-3}$.

$$\begin{array}{r}
 19. \quad 5x^4 - 11x^2 + 6 \overline{) 5x^4 - 2x^2 - 3} \quad (1 \\
 \underline{5x^4 - 11x^2 + 6} \\
 9 \overline{) 9x^2 - 9} \\
 \underline{x^2 - 1} \quad 5x^4 - 11x^2 + 6(5x^2 - 6) \\
 \underline{5x^4 - 5x^2} \\
 - 6x^2 + 6 \\
 - 6x^2 + 6
 \end{array}$$

Dividing numerator and denominator by $x^2 - 1$, we obtain $\frac{5x^2+8}{5x^2-6}$.

$$\begin{array}{r}
 20. \quad x^3 - 10x + 21 \overline{) x^3 - 11x + 6} \quad (x + 10 \\
 \underline{x^3 - 10x^2 + 21x} \\
 10x^2 - 82x + 6 \\
 \underline{10x^2 - 100x + 210} \\
 68 \overline{) 68x - 204} \\
 \underline{x - 8} (x - 7 \\
 \underline{x^3 - 8x} \\
 \underline{- 7x + 21} \\
 \underline{- 7x + 21}
 \end{array}$$

Dividing numerator and denominator by $x - 8$, we obtain $\frac{x - 7}{x^2 + 8x - 2}$.

$$\begin{array}{r}
 21. \quad x^3 - 7x + 6 \overline{) x^3 - 2x^2 - 8x - 96} \quad (x + 5 \\
 \underline{x^3 - 7x^2 + 6x} \\
 5x^2 - 14x - 96 \\
 \underline{5x^2 - 35x + 80} \\
 21 \overline{) 21x - 126} \\
 \phantom{21 \overline{) 21x - 126}} \underline{x - 6} (x - 1 \\
 \phantom{21 \overline{) 21x - 126}} \underline{x^2 - 6x} \\
 \phantom{21 \overline{) 21x - 126}} \underline{- x + 6} \\
 \phantom{21 \overline{) 21x - 126}} \underline{- x + 6}
 \end{array}$$

Dividing numerator and denominator by $x - 6$, we obtain $\frac{x - 1}{x^2 + 4x + 16}$.

$$\begin{array}{r}
 22. \quad 4x^2 - 5x + 1 \overline{) 12x^3 - 12x^2 + 4x - 4} \quad (3x \\
 \underline{12x^3 - 15x^2 + 3x} \\
 3x^2 + x - 4 \\
 \underline{4} \\
 \underline{12x^2 + 4x - 16} \quad (3 \\
 \underline{12x^2 - 15x + 3} \\
 \underline{19 \overline{) 19x - 19}} \\
 \phantom{19 \overline{) 19x - 19}} \underline{x - 1} (4x - 1 \\
 \phantom{19 \overline{) 19x - 19}} \underline{4x^2 - 4x} \\
 \phantom{19 \overline{) 19x - 19}} \underline{- x + 1} \\
 \phantom{19 \overline{) 19x - 19}} \underline{- x + 1}
 \end{array}$$

Dividing numerator and denominator by $x - 1$, we obtain $\frac{4x - 1}{12x^3 + 4}$.

$$\begin{array}{r}
 23. \quad x^3 + 7x^2 + 17x + 15 \overline{) x^3 + 8x^2 + 19x + 12} \quad (1 \\
 \underline{x^3 + 7x^2 + 17x + 15} \\
 x^2 + 2x - 8 \quad \overline{) x^3 + 7x^2 + 17x + 15} \quad (x + 5 \\
 \underline{x^3 + 2x^2 - 8x} \\
 5x^2 + 20x + 15 \\
 \underline{5x^2 + 10x - 15} \\
 10 \overline{) 10x + 80} \\
 \underline{10x + 80} \\
 x + 8 \overline{) x^2 + 2x - 8} \quad (x - 1 \\
 \underline{x^2 + 8x} \\
 -x - 8 \\
 \underline{-x - 8}
 \end{array}$$

Dividing numerator and denominator by $x + 3$, we obtain $\frac{x^2 + 4x + 5}{x^2 + 5x + 4}$.

$$\begin{array}{r}
 24. \quad x^3 + 1 \overline{) x^3 - 4x^2 + 5} \quad (1 \\
 \underline{x^3} \quad \quad \quad \underline{+ 1} \\
 -4 \overline{) -4x^2 + 4} \\
 \underline{-4x^2 + 4} \\
 x^3 - 1 \overline{) x^3 + 1} \quad (x \\
 \underline{x^3 - x} \\
 x + 1 \overline{) x^3 - 1} \quad (x - 1 \\
 \underline{x^3 + x} \\
 -x - 1 \\
 \underline{-x - 1}
 \end{array}$$

Dividing numerator and denominator by $x + 1$, we find $\frac{x^2 - 5x + 5}{x^2 - x + 1}$.

$$\begin{array}{r}
 25. \quad x^3 - 4x^2 + 2x + 8 \overline{) 2x^4 - 9x^3 + 12x^2 - 7} \quad (2x - 1 \\
 \underline{2x^4 - 8x^3 + 4x^2 + 6x} \\
 -x^3 + 8x^2 - 6x - 7 \\
 \underline{-x^3 + 4x^2 - 2x - 8} \\
 4 \overline{) 4x^2 - 4x - 4} \\
 \underline{4x^2 - 4x - 4} \\
 x^3 - x - 1 \overline{) x^3 - 4x^2 + 2x + 8} \quad (x - 8 \\
 \underline{x^3 - x^2 - x} \\
 -3x^2 + 8x + 8 \\
 \underline{-3x^2 + 8x + 8}
 \end{array}$$

Dividing numerator and denominator by $x^3 - x - 1$, we find $\frac{x - 8}{2x^2 - 7x + 7}$.

$$\begin{array}{r}
 26. \quad x^3 - ax^2 - a^2x - 2a^3 \quad x^4 + a^2x^2 + a^4 \quad (x + a \\
 \hline
 x^4 - ax^3 - a^2x^2 - 2a^3x \\
 \hline
 ax^3 + 2a^2x^2 + 2a^3x + a^4 \\
 \hline
 ax^3 - a^2x^2 - a^3x - 2a^4 \\
 \hline
 3a^2) 3a^2x^2 + 3a^3x + 3a^4 \\
 \hline
 x^2 + ax + a^3) x^3 - ax^2 - a^2x - 2a^3(x - 2a \\
 \hline
 x^3 + ax^2 + a^3x \\
 \hline
 -2ax^2 - 2a^2x - 2a^3 \\
 \hline
 -2ax^2 - 2a^2x - 2a^3
 \end{array}$$

Dividing numerator and denominator by $x^2 + ax + a^2$, we obtain $\frac{x - 2a}{x^2 - ax + a^2}$.

27. Divide numerator by 3; then

$$\begin{array}{r}
 x^2 - 4) x^3 - 2x^2 + 5x - 10(x - 2 \\
 \hline
 x^3 - 4x^2 \\
 \hline
 -2x^2 + 9x - 10 \\
 \hline
 -2x^2 + 8 \\
 \hline
 9) 9x - 18 \\
 \hline
 x - 2) x^2 - 4 \quad (x + 2 \\
 \hline
 x^2 - 2x \\
 \hline
 2x - 4 \\
 \hline
 2x - 4
 \end{array}$$

Dividing numerator and denominator by $x - 2$, we find $\frac{3(x + 2)}{x^2 + 5}$.

$$\begin{array}{r}
 28. \quad 4x^3 - 4x - 3) 6x^2 - 5x - 6(2 \\
 \hline
 8x^2 - 8x - 6 \\
 \hline
 -x) - 2x^2 + 3x \\
 \hline
 2x - 3) 4x^2 - 4x - 3(2x + 1 \\
 \hline
 4x^2 - 6x \\
 \hline
 2x - 3 \\
 \hline
 2x - 3
 \end{array}$$

Dividing numerator and denominator by $2x - 3$, we obtain $\frac{2x + 1}{3x + 2}$.

$$\begin{array}{r}
 29. \quad 12x^2 + 7xy + y^2) 28x^2 + 3xy - y^2(-1 \\
 \hline
 -12x^2 - 7xy - y^2 \\
 \hline
 10x) 40x^2 + 10xy \\
 \hline
 4x + y) 12x^2 + 7xy + y^2(3x + y \\
 \hline
 12x^2 + 3xy \\
 \hline
 4xy + y^3 \\
 \hline
 4xy + y^3
 \end{array}$$

Dividing numerator and denominator by $4x + y$, we obtain $\frac{3x + y}{7x - y}$.

$$\begin{array}{r}
 30. \quad 12x^2 + x - 1 \quad 15x^2 + 8x + 1 \quad (-1 \\
 \quad \quad \quad - 12x^2 - \quad x + 1 \\
 \hline
 \quad \quad \quad 9x \quad 27x^2 + 9x \\
 \quad \quad \quad \quad 3x + 1 \quad 12x^2 + \quad x - 1 \quad (4x - 1 \\
 \quad \quad \quad \quad \quad 12x^2 + 4x \\
 \quad \quad \quad \quad \quad \quad - 3x - 1 \\
 \quad \quad \quad \quad \quad \quad - 3x - 1 \\
 \hline
 \end{array}$$

Dividing numerator and denominator by $3x + 1$, we obtain $\frac{4x-1}{5x+1}$.

31. Divide denominator by x ; then

$$\begin{array}{r}
 6x^2 - 17x + 12 \quad 8x^2 - 6x - 9 \\
 \quad \quad \quad \quad \quad 3 \\
 \hline
 \quad \quad \quad 24x^2 - 18x - 27 \quad (4 \\
 \quad \quad \quad 24x^2 - 68x + 48 \\
 \hline
 \quad \quad \quad 25 \quad 50x - 75 \\
 \quad \quad \quad \quad 2x - 3 \quad 6x^2 - 17x + 12 \quad (8x - 4 \\
 \quad \quad \quad \quad \quad 6x^2 - 9x \\
 \quad \quad \quad \quad \quad \quad - 8x + 12 \\
 \quad \quad \quad \quad \quad \quad - 8x + 12 \\
 \hline
 \end{array}$$

Dividing numerator and denominator by $2x - 3$, we obtain $\frac{4x+8}{3x^2-4x}$.

$$\begin{array}{r}
 32. \quad x^2 + 4x + 4 \quad x^2 + 8 \quad (x - 4 \\
 \quad \quad \quad x^2 + 4x^2 + 4x \\
 \quad \quad \quad \quad - 4x^2 - 4x + 8 \\
 \quad \quad \quad \quad - 4x^2 - 16x - 16 \\
 \hline
 \quad \quad \quad 12 \quad 12x + 24 \\
 \quad \quad \quad \quad x + 2 \quad x^2 + 4x + 4 \quad (x + 2 \\
 \quad \quad \quad \quad \quad x^2 + 2x \\
 \quad \quad \quad \quad \quad \quad 2x + 4 \\
 \quad \quad \quad \quad \quad \quad 2x + 4 \\
 \hline
 \end{array}$$

Dividing numerator and denominator by $x + 2$, we obtain $\frac{x+2}{x^2-2x+4}$.

EXERCISE XIV.

$$\begin{array}{r}
 38. \quad x^3 + 6x^2 + 11x + 12 \overline{) x^3 + 7x^2 + 16x + 16} \\
 \underline{x^3 + 6x^2 + 11x + 12} \\
 x^2 + 5x + 4 \\
 \underline{x^3 + 6x^2 + 11x + 12} \\
 x^2 + 5x^2 + 4x \\
 \underline{x^2 + 7x + 12} \\
 x^2 + 5x + 4 \\
 \underline{2) 2x + 8} \\
 x + 4 \\
 \underline{x^2 + 5x + 4} \\
 x^2 + 4x \\
 \underline{x + 4} \\
 x + 4
 \end{array}$$

Dividing numerator and denominator by $x + 4$, we obtain $\frac{x^2 + 3x + 4}{2(x^2 + 2x + 8)}$.

$$\begin{array}{r}
 34. \quad \begin{array}{r} 6x^2 - 13x + 6 \end{array} \overline{) 9x^3 - 13x + 6} \begin{array}{l} 1 \\ 6x^2 - 13x + 6 \end{array} \\
 \begin{array}{r} 3x^2 \end{array} \overline{) 9x^3 - 6x^2} \\
 \begin{array}{r} 3x - 2 \end{array} \overline{) 6x^2 - 13x + 6} \begin{array}{l} 2x - 3 \\ 6x^2 - 4x \\ - 9x + 6 \\ - 9x + 6 \end{array}
 \end{array}$$

Dividing numerator and denominator by $3x - 2$, we obtain $\frac{2x-3}{3x^2+2x-3}$.

$$\begin{array}{r}
 35. \quad \text{Divide denominator by } 3y; \text{ then} \\
 \begin{array}{r} 3x^2 - 13x + 14 \end{array} \overline{) 14x^2 - 34x + 12} \begin{array}{l} 3 \\ 42x^2 - 102x + 36 \end{array} \begin{array}{l} 14 \\ 42x^2 - 182x + 196 \end{array} \\
 \begin{array}{r} 80 \end{array} \overline{) 80x - 160} \\
 \begin{array}{r} x - 2 \end{array} \overline{) 8x^2 - 13x + 14} \begin{array}{l} 3x - 7 \\ 3x^2 - 6x \\ - 7x + 14 \\ - 7x + 14 \end{array}
 \end{array}$$

Dividing numerator and denominator by $x - 2$, we find $\frac{14x-6}{3y(3x-7)}$.

$$\begin{array}{r}
 36. \quad \begin{array}{r} 3x^2 + x - 4 \end{array} \overline{) 2x^3 - 3x^2 + 1} \begin{array}{l} 3 \\ 6x^3 - 9x^2 + 3 \end{array} \begin{array}{l} 2x \\ 6x^3 + 2x^2 - 8x \end{array} \\
 \begin{array}{r} -11x^2 + 8x + 3 \end{array} \\
 \begin{array}{r} -33x^2 + 24x + 9 \end{array} \begin{array}{l} (-11 \\ -33x^2 - 11x + 44 \end{array} \\
 \begin{array}{r} 35 \end{array} \overline{) 35x - 85} \\
 \begin{array}{r} x - 1 \end{array} \overline{) 3x^2 + x - 4} \begin{array}{l} 3x + 4 \\ 3x^2 - 3x \\ 4x - 4 \\ 4x - 4 \end{array}
 \end{array}$$

Dividing numerator and denominator by $x - 1$, we find $\frac{3x+4}{2x^2-x-1}$.

$$\begin{array}{r}
 37. \quad \begin{array}{r} 2x^3 - x^2 - x + 2 \end{array} \overline{) 6x^3 - 5x^2 + 4} \begin{array}{l} 3 \\ 6x^3 - 3x^2 - 3x + 6 \end{array} \\
 \begin{array}{r} -2x^2 + 3x - 2 \end{array} \overline{) 2x^3 - x^2 - x + 2} \begin{array}{l} (-x - 1 \\ 2x^3 - 3x^2 + 2x \\ 2x^2 - 3x + 2 \\ 2x^2 - 3x + 2 \end{array}
 \end{array}$$

Dividing numerator and denominator by $2x^2 - 3x + 2$, we find $\frac{x+1}{3x+2}$.

$$\begin{array}{r}
 40. \quad 3p^2 - 2pq - q^2) \quad 4p^3 - 2p^2q - 3pq^2 + \frac{q^3}{3} \\
 \underline{12p^3 - 6p^2q - 9pq^2 + 3q^3} \quad (4p \\
 12p^3 - 8p^2q - 4pq^2 \\
 \underline{2p^2q - 5pq^2 + 3q^3} \\
 6p^2q - 15pq^2 + 9q^3 \quad (2q \\
 6p^2q - 4pq^2 - 2q^3 \\
 \underline{-11q^3} \quad -11pq^2 + 11q^3 \\
 \frac{p-q}{3p^2-3pq} \quad 3p^2 - 2pq - q^2 (3p+q \\
 \frac{pq-q^2}{pq-q^2}
 \end{array}$$

Dividing numerator and denominator by $p - q$, we obtain

$$\frac{3p+q}{q(4p^2+2pq-q^2)}.$$

41. Divide numerator by x^2 ; then

$$\begin{array}{r}
 3x^2 - 4x - 4) \quad 4x^3 - 11x^2 + 8x - 4 \\
 \underline{12x^3 - 33x^2 + 24x - 12} \quad (4x+3 \\
 12x^3 - 16x^2 - 16x \\
 \underline{-17x^2 + 40x - 12} \\
 9x^2 - 12x - 12 \\
 \underline{-26x^2 + 52x} \\
 x-2) \quad 3x^2 - 4x - 4 \quad (3x+2 \\
 \underline{3x^2 - 6x} \\
 2x - 4 \\
 \underline{2x - 4}
 \end{array}$$

Dividing numerator and denominator by $x - 2$, we find $\frac{x^2(3x+2)}{4x^2-3x+2}$.

49. Divide numerator by x ; then

$$\begin{array}{r} 4x^3 - 6x^2 - 4x + 8 \quad 6x^3 + x^2 - 1 \end{array}$$

$$\frac{12x^3 + 2x^2 - 12x + 9}{12x^3 - 18x^2 - 12x + 9} = \frac{2(8)}{20x^2 + 12x - 11}$$

$$\begin{array}{r} 4x^3 - 6x^2 - 4x + 8 \\ 5 \end{array}$$

$$\begin{array}{r} 20x^3 - 90x^2 - 20x + 15(x) \\ 20x^3 + 12x^2 - 11x \\ \hline - 42x^2 - 9x + 15 \\ - 10 \end{array}$$

$$\begin{array}{r} 420x^3 + 90x - 150(21) \\ 420x^3 + 252x - 231 \\ \hline - 81 - 162x + 81 \end{array}$$

$$\begin{array}{r} 2x - 1 \\ 20x^3 + 12x - 11(10x + 11) \\ \hline 20x^3 - 10x \\ \hline 22x - 11 \\ \hline 22x - 11 \end{array}$$

Dividing numerator and denominator by $2x - 1$, we obtain $\frac{(3x^2 + 2x + 1)x}{2x^2 - 2x - 3}$.

43. Divide denominator by x ; then

$$\begin{array}{r} 4x^3 - 9x^2 + 14x - 3 \quad 3x^4 - 6x^3 + 11x^2 - 4x + 6 \quad (-2 - 8x \\ - 8x^3 + 18x^2 - 28x + 6 \\ \hline 3x^4 + 2x^3 - 7x^2 + 24x \\ - 32x^4 + 72x^3 - 112x^2 + 24x \\ \hline 35x^3 35x^4 - 70x^3 + 105x^2 \\ x^2 - 2x + 3 \quad 4x^3 - 9x^2 + 14x - 3 \quad (4x - 1 \\ 4x^3 - 8x^2 + 12x \\ - x^2 + 2x - 3 \\ - x^2 + 2x - 3 \\ \hline 3x^2 + 2 \end{array}$$

Dividing numerator and denominator by $x^2 - 2x + 3$, we find $\frac{3x^2 + 2}{(4x - 1)x}$.

$$\begin{array}{r} 44. \quad 2x^4 + x^3 - 2x^2 - 4x - 3 \quad 2x^5 - x^4 - x^3 - x^2 - x - 3 \quad (x - 1 \\ 2x^5 + x^4 - 2x^3 - 4x^2 - 3x \\ \hline - 2x^4 + x^3 + 3x^2 + 2x - 3 \\ - 2x^4 - x^3 + 2x^2 + 4x + 3 \\ \hline 2x^3 + x^2 - 2x - 6 \quad 2x^4 + x^3 - 2x^2 - 4x - 3 \quad (x \\ 2x^4 + x^3 - 2x^2 - 6x \\ \hline 2x - 3 \quad 2x^3 + x^2 - 2x + 2 \\ 2x^3 - 3x^2 \\ \hline 4x^2 - 2x - 6 \\ 4x^2 - 6x \\ \hline 4x - 6 \\ 4x - 6 \end{array}$$

Dividing numerator and denominator by $2x - 3$, we find $\frac{x^2 + 2x^2 + 2x + 1}{x^4 + x^3 + x^2 + x + 1}$.

D. A. K.

45. Divide numerator by 2; then

$$\frac{2x^4 - 3x^2y + 8xy^2 + 4y^4}{8y} \cdot \frac{10x^4 + 17x^2y - 11xy^2 - 4y^4}{10x^4 - 15x^2y + 45xy^2 + 20y^4}$$

$$\frac{4x^3 - 7xy^2 - 3y^3}{2} \cdot \frac{2x^4 - 3x^2y + 8xy^2 + 4y^4}{2}$$

$$\frac{4x^4 - 6x^2y + 18xy^2 + 8y^4}{4x^4 - 7x^2y^2 - 8xy^3} \cdot \frac{8y^4}{2}$$

$$\frac{-6x^2y + 7x^2y^2 + 21xy^2 + 8y^4}{-12x^3y + 14x^2y^2 + 42xy^3 + 16y^4} \cdot \frac{8y^4}{2}$$

$$\frac{-12x^3y + 14x^2y^2 + 42xy^3 + 16y^4}{-12x^3y + 21xy^3 + 7y^4} \cdot \frac{8y^4}{2}$$

$$\frac{2x^4 - 3x^2y + 8xy^2 + 4y^4}{2x^4 + 3x^2y + x^2y^2} \cdot \frac{2x^4 - 3x^2y + 8xy^2 + 4y^4}{2x^4 + 3x^2y + x^2y^2}$$

$$\frac{-6x^2y - x^2y^2 + 9xy^3 + 4y^4}{-6x^2y - 9x^2y^2 - 8xy^3}$$

$$\frac{8x^2y^2 + 12xy^3 + 4y^4}{8x^2y^2 + 12xy^3 + 4y^4}$$

Dividing numerator and denominator by $2x^2 + 8xy + y^2$, we obtain $\frac{2x^2 - 6xy + 8y^2}{6x^2 + xy - 4y^2}$.

EXERCISE XV.

$$\begin{array}{r}
 1. \quad x^2 - 6x + 8 \overline{) x^2 - 8x + 16} \quad (1 \\
 \underline{x^2 - 6x + 8} \\
 -2x + 8 \\
 x - 4 \overline{) x^2 - 6x + 8} \quad (x - 2 \\
 \underline{x^2 - 4x} \\
 -2x + 8 \\
 \underline{-2x + 8}
 \end{array}$$

Dividing the two expressions by $x - 4$, we obtain $x - 2$ and $x - 4$; hence L. C. M. is $(x - 2)(x - 4)^2$.

$$\begin{array}{r}
 2. \quad x^2 + 5x - 84 \overline{) x^2 + 21x + 108} \quad (1 \\
 \underline{x^2 + 5x - 84} \\
 16x + 192 \\
 x + 12 \overline{) x^2 + 5x - 84} \quad (x - 7 \\
 \underline{x^2 + 12x} \\
 -7x - 84 \\
 \underline{-7x - 84}
 \end{array}$$

Dividing the two expressions by $x + 12$, we obtain $x - 7$ and $x + 9$; hence L. C. M. is $(x + 12)(x - 7)(x + 9)$.

$$\begin{array}{r}
 3. \quad 3x^2 - 7x + 4 \overline{) 6x^2 - 10x + 4} \quad (2 \\
 \underline{6x^2 - 14x + 8} \\
 4x - 4 \\
 x - 1 \overline{) 3x^2 - 7x + 4} \quad (3x - 4 \\
 \underline{3x^2 - 3x} \\
 -4x + 4 \\
 \underline{-4x + 4}
 \end{array}$$

Dividing the two expressions by $x - 1$, we obtain $3x - 4$ and $6x - 4$; hence L. C. M. is $(x - 1)(3x - 4)(6x - 4)$.

$$\begin{array}{r}
 4. \quad 2x^2 - 7x + 5 \overline{) 6x^2 - 23x + 20} \quad (3 \\
 \underline{6x^2 - 21x + 15} \\
 -2x + 5 \\
 2x - 5 \overline{) 2x^2 - 7x + 5} \quad (-x + 1 \\
 \underline{2x^2 - 5x} \\
 -2x + 5 \\
 \underline{-2x + 5}
 \end{array}$$

Dividing the two expressions by $2x - 5$, we obtain $x - 1$ and $3x - 4$; hence L. C. M. is $(2x - 5)(x - 1)(3x - 4)$.

$$\begin{array}{r}
 5. \quad 2x^3 + 13x + 15 \overline{) 8x^3 + 10x - 3} \quad (4 \\
 \underline{8x^3 + 52x + 60} \\
 -21 \overline{) -42x - 63} \\
 \underline{2x + 3} \quad 2x^3 + 13x + 15 \overline{) (x + 5} \\
 \underline{2x^3 + 3x} \\
 10x + 15 \\
 \underline{10x + 15} \\
 0
 \end{array}$$

Dividing the two expressions by $2x + 3$, we obtain $x + 5$ and $4x - 1$; hence the L. C. M. is $(2x + 3)(x + 5)(4x - 1)$.

$$\begin{array}{r}
 6. \quad x^3 - 18xy + 32y^3 \overline{) x^3 - 9xy + 14y^3} \quad (1 \\
 \underline{x^3 - 18xy + 32y^3} \\
 9y \overline{) 9xy - 18y^3} \\
 \underline{x - 2y} \quad x^3 - 18xy + 32y^3 \overline{) (x - 16y} \\
 \underline{x^3 - 2xy} \\
 -16xy + 32y^3 \\
 \underline{-16xy + 32y^3} \\
 0
 \end{array}$$

Dividing the two expressions by $x - 2y$, we obtain $x - 16y$ and $x - 7y$; hence L. C. M. is $(x - 2y)(x - 16y)(x - 7y)$.

$$\begin{array}{r}
 7. \quad 2x^3 + 3x - 20 \overline{) 6x^3 - 25x^2 + 21x + 10} \quad (3x - 17 \\
 \underline{6x^3 + 9x^2 - 60x} \\
 -34x^2 + 81x + 10 \\
 \underline{-34x^2 - 51x + 340} \\
 66 \overline{) 132x - 330} \\
 \underline{2x - 5} \quad 2x^3 + 3x - 20 \overline{) (x + 4} \\
 \underline{2x^3 - 5x} \\
 8x - 20 \\
 \underline{8x - 20} \\
 0
 \end{array}$$

Dividing both expressions by $2x - 5$, we obtain $x + 4$ and $3x^2 - 5x - 2$; hence L. C. M. is $(2x - 5)(x + 4)(3x^2 - 5x - 2)$.

$$8. \quad \frac{x^4 - 8x^3 + 16}{x^4 - 8x^3} \frac{x^4 - 7x^3 + 28x - 16}{x^4 - 8x^3} + 16$$

$$- \frac{7x^3 + 8x^2 + 28x - 32}{x^4 - 8x^3} \frac{x^4 - 8x^3 + 16}{x^4 - 8x^3}$$

$$\frac{7x^4 - 56x^3 + 112}{7x^4 - 8x^3 - 28x^2 + 32x} \frac{(-x)}{8x^3 - 28x^2 - 32x + 112}$$

$$\frac{56x^3 - 196x^2 - 224x + 784}{56x^3 - 64x^2 - 224x + 256} \frac{(-8)}{-132 - 132x^2 + 528}$$

$$\frac{x^3 - 4}{7x^3} \frac{7x^3 - 8x^2 - 28x + 32}{7x^3 - 28x}$$

$$\frac{-8x^2 + 32}{-8x^2 + 32}$$

Dividing both expressions by $x^3 - 4$, we obtain $x^3 - 4$ and $x^3 - 7x + 4$; hence L. C. M. is $(x^3 - 4)^2 (x^3 - 7x + 4)$.

$$\begin{array}{r}
 9. \quad x^3 - 8x + 7 \overline{) 2x^3 - 18x^2 + 46x - 30} \quad (2 \\
 \underline{2x^3} \\
 -2 \\
 \underline{-2} \\
 9x^2 - 81x + 22 \quad x^3 - 8x + 7 \\
 \underline{9x^2 - 81x + 22} \\
 7
 \end{array}$$

$$\begin{array}{r}
 \quad \frac{9x^2 - 72x + 63}{9x^2 - 81x + 22} \left(\frac{x}{9} \right. \\
 \quad \underline{81x^2 - 94x + 63} \\
 \quad 9
 \end{array}$$

$$\begin{array}{r}
 \quad \frac{279x^3 - 846x + 567}{279x^3 - 961x + 682} \left(\frac{31}{115} \right) \\
 \quad \underline{115} \overline{) 115x - 115} \\
 \quad x - 1 \quad 9x^2 - 81x + 22 \quad (9x - 23 \\
 \quad \underline{9x^2 - 9x} \\
 \quad -22x + 23 \\
 \quad \underline{-22x + 22} \\
 \quad 1
 \end{array}$$

Dividing the two expressions by $x - 1$, we obtain $x^2 + x - 7$ and $2x^2 - 16x + 30$; hence L. c. m. is $2(x - 1)(x^2 + x - 7)(x^2 - 8x + 15)$.

$$\begin{array}{r}
 10. \quad x^3 - 15x + 36 \overline{) x^3 - 3x^2 - 2x + 6(x+12)} \\
 \underline{x^3 - 15x^2 + 36x} \\
 12x^2 - 88x + 6 \\
 \underline{12x^2 - 180x + 432} \\
 142 \overline{) 142x - 426} \\
 \underline{x-3} x^2 - 15x + 36(x-12) \\
 \underline{x^2 - 3x} \\
 \underline{-12x + 36} \\
 \underline{-12x + 36}
 \end{array}$$

Dividing the two expressions by $x-3$, we obtain $x-12$ and x^2-2 ; hence l. c. m. is $(x-3)(x-12)(x^2-2)$.

$$\begin{array}{r}
 11. \quad 10x^3 + 49x - 33 \overline{) 10x^3 + 29x^2 + 14x - 21(x-2)} \\
 \underline{10x^3 + 49x^2 - 33x} \\
 -20x^2 + 47x - 21 \\
 \underline{-20x^2 - 98x + 66} \\
 29 \overline{) 145x - 87} \\
 \underline{5x-3} 10x^2 + 49x - 33(2x+11) \\
 \underline{10x^2 - 6x} \\
 \underline{55x - 33} \\
 \underline{55x - 33}
 \end{array}$$

Dividing both expressions by $5x-3$, we obtain $2x+11$ and $2x^2+7x+7$; hence l. c. m. is $(5x-3)(2x+11)(2x^2+7x+7)$.

$$\begin{array}{r}
 12. \quad x^3 - x^2 - 7x + 15 \overline{) x^3 - 3x + 9(1)} \\
 \underline{x^3 - x^2 - 7x + 15} \\
 2 \overline{) 2x^2 + 4x - 6} \\
 \underline{x^2 + 2x - 3} x^3 - x^2 - 7x + 15(x-3) \\
 \underline{x^3 + 2x^2 - 3x} \\
 \underline{-3x^2 - 4x + 15} \\
 \underline{-3x^2 - 6x + 9} \\
 \underline{2) 2x + 6} \\
 \underline{x+3} x^2 + 2x - 3(x-1) \\
 \underline{x^2 + 3x} \\
 \underline{-x-3} \\
 \underline{-x-3}
 \end{array}$$

Dividing both expressions by $x+3$, we obtain as l. c. m.

$$(x+3)(x^2-4x+5)(x^2-2x+3).$$

$$\begin{array}{r}
 13. \quad x^3 + x - 12 \big) x^3 - 5x^2 + 7x - 3(x - 6) \\
 \underline{x^3 + x^2 - 12x} \\
 -6x^2 + 19x - 3 \\
 \underline{-6x^2 - 6x + 72} \\
 25 \big) 25x - 75 \\
 \underline{25x - 75} \\
 0
 \end{array}$$

Dividing the two expressions by $x - 3$, we obtain as L. C. M.

$$\begin{array}{r}
 (x - 3)(x + 4)(x^2 - 2x + 1). \\
 14. \quad x^3 - 8x - 28 \big) x^3 - 5x^2 - 11x - 21(x - 2) \\
 \underline{x^3 - 3x^2 - 28x} \\
 -2x^2 + 17x - 21 \\
 \underline{-2x^2 + 6x + 56} \\
 11 \big) 11x - 77 \\
 \underline{11x - 77} \\
 0
 \end{array}$$

Dividing the two expressions by $x - 7$, we obtain as L. C. M.

$$\begin{array}{r}
 (x - 7)(x + 4)(x^2 + 2x + 3). \\
 15. \quad x^3 - 7x + 10 \big) 4x^3 - 25x^2 + 20x + 25(4x + 3) \\
 \underline{4x^3 - 28x^2 + 40x} \\
 3x^2 - 20x + 25 \\
 \underline{3x^2 - 21x + 30} \\
 x - 5 \big) x^2 - 7x + 10(x - 2) \\
 \underline{x^2 - 5x} \\
 -2x + 10 \\
 \underline{-2x + 10} \\
 0
 \end{array}$$

Dividing both expressions by $x - 5$, we obtain as L. C. M.

$$\begin{array}{r}
 (x - 5)(x - 2)(4x^2 - 5x - 5). \\
 16. \quad x^3 - 3x - 70 \big) x^3 - 39x + 70(x + 3) \\
 \underline{x^3 - 3x^2 - 70x} \\
 3x^2 + 31x + 70 \\
 \underline{3x^2 - 9x - 210} \\
 40 \big) 40x + 280 \\
 \underline{40x + 280} \\
 0
 \end{array}$$

Dividing the two expressions by $x + 7$, we obtain as L. C. M.

$$(x + 7)(x - 10)(x^2 - 7x + 10).$$

$$\begin{array}{r}
17. \quad 3x^3 - 13x^2 + 23x - 21 \mid 6x^3 + x^2 - 44x + 21 \quad (2) \\
\underline{6x^3 - 26x^2 + 46x - 42} \\
9 \mid 27x^2 - 90x + 63 \\
\underline{3x^3 - 10x + 7} \quad 3x^3 - 13x^2 + 23x - 21 \mid (x - 1) \\
\underline{3x^3 - 10x^2 + 7x} \\
-3x^2 + 16x - 21 \\
\underline{-3x^2 + 10x - 7} \\
2 \mid 6x - 14 \\
\underline{3x - 7} \quad 3x^3 - 10x + 7 \mid (x - 1) \\
\underline{3x^3 - 7x} \\
-3x + 7 \\
\underline{-3x + 7} \\
0
\end{array}$$

19.

$$\begin{array}{r}
 6x^3 - 11x + 5 \quad 3x^3 - 2x^2 - 1 \\
 \quad \quad \quad 2 \\
 \hline
 6x^3 - 4x^2 - 2(x \\
 6x^3 - 11x^2 + 5x \\
 \hline
 7x^2 - 5x - 2 \\
 \quad \quad \quad 6 \\
 \hline
 42x^2 - 30x - 12 \quad (7 \\
 42x^2 - 77x + 35 \\
 \hline
 47 \quad 47x - 47 \\
 \quad \quad \quad x - 1 \quad 6x^2 - 11x + 5(6x - 5 \\
 \quad \quad \quad 6x^2 - 6x \\
 \quad - 5x + 5 \\
 \quad - 5x + 5 \\
 \hline
 \quad -
 \end{array}$$

Dividing the two expressions by $x - 1$, we obtain as L. C. M.,

$$(x - 1)(6x - 5)(3x^2 + x + 1).$$

20. Divide the first expression by 2, and the second by x ;

$$\begin{array}{r}
 2x^3 - 7x - 15 \quad 6x^2 + 7x - 3(3 \\
 6x^2 - 21x - 45 \\
 \hline
 14 \quad 28x + 42 \\
 \quad \quad \quad 2x + 3 \quad 2x^2 - 7x - 15(x - 5 \\
 \quad \quad \quad 2x^2 + 3x \\
 \quad - 10x - 15 \\
 \quad - 10x - 15 \\
 \hline
 \quad -
 \end{array}$$

Dividing the two expressions by $2x + 3$, we obtain as L. C. M.

$$2x(2x + 3)(x - 5)(3x - 1).$$

$$\begin{array}{l}
 21. \quad \frac{20x^4 + x^3 - 1}{x} \frac{25x^4 + 5x^3 - x - 1}{5x^4 + 5x^3 - x^2 - x} \frac{20x^4 + x^2 - 1}{5x^3 + 5x^2 - x - 1} \frac{(4x - 4)}{20x^4 + 20x^3 - 4x^3 - 4x} \\
 \quad \frac{-20x^3 + 5x^2 + 4x - 1}{-20x^3 - 20x^3 + 4x + 4} \frac{5}{5} \frac{25x^3}{5x^3 - 1} \frac{5x^3 + 5x^2 - x - 1}{5x^3} (x + 1) \\
 \quad \frac{5x^3}{5x^3} \frac{-1}{-1}
 \end{array}$$

Dividing the two expressions by $5x^2 - 1$, we obtain as L. C. M. $(5x^2 - 1)(4x^2 + 1)(5x^2 + x + 1)$.

$$\begin{array}{l}
 22. \quad \frac{6x^3 + x^2 - 1}{-18x^3 - 8x^2 + 3} \frac{4x^3 - 6x^2 - 4x + 3}{22x^3 - 3x^2 - 4x} \frac{22x^2 - 3x - 4}{8x^2 - 6x + 1} \frac{22x^2 - 3x - 4}{-32x^3 + 24x - 4} \\
 \quad \frac{24x^4 + 4x^3 - 4x}{-3x^2} \frac{24x^4 + 18x^3 - 8x^2}{8x^2 - 6x + 1} \frac{27x}{27x} \frac{54x^3 - 27x}{2x - 1} \frac{8x^2 - 6x + 1}{8x^2 - 4x} \frac{-2x + 1}{-2x + 1} \\
 \quad \frac{-2x + 1}{-2x + 1}
 \end{array}$$

Dividing both expressions by $2x - 1$, we obtain as L. C. M. $x(2x - 1)(2x^2 - 2x - 8)(3x^2 + 2x + 1)$.

$$23. \quad \frac{a^4 - 11a^3 + 35a^2 - 78a + 24}{8} \frac{3a^4 - 17a^3 + 49a^2 - 67a + 8}{16a^3 - 56a^2 + 152a - 64} (3a^4 - 88a^3 + 105a^2 - 219a + 72)$$

$$\frac{2a^3 - 7a^2 + 19a - 8}{2} \frac{a^4 - 11a^3 + 35a^2 - 78a + 24}{2}$$

$$\frac{2a^4 - 22a^3 + 70a^2 - 146a + 48}{2a^4 - 7a^3 + 19a^2 - 8a} (a - 6)$$

$$\frac{-15a^3 + 51a^2 - 138a + 48}{-12a^3 + 42a^2 - 114a + 48}$$

$$\frac{-8a}{a^3 - 3a^2 + 9a^2 - 24a}$$

$$\frac{a^3 - 3a + 8}{2a^3 - 7a^2 + 19a - 8} \frac{2a^3 - 7a^2 + 19a - 8}{2a^3 - 6a^2 + 16a} (2a - 1)$$

$$\frac{-a^3 + 3a - 8}{-a^3 + 3a - 8}$$

Dividing both expressions by $a^3 - 3a + 8$, we obtain as L. C. M. $(a^3 - 3a + 8)(a^3 - 8a + 8)(3a^3 - 8a + 1)$.

24.

$$x^2 + 4x + 3) x^5 + 5x^3 + 6 \quad (x^3 - 4x^2 + 18x - 60)$$

$$\frac{x^5 + 4x^4 + 3x^3}{-4x^4 + 2x^3 + 6}$$

$$\frac{-4x^4 + 2x^3 + 6}{-4x^4 - 16x^3 - 12x^2}$$

$$\frac{18x^3 + 12x^2 + 6}{18x^3 + 72x^2 + 54x}$$

$$\frac{-60x^2 - 54x + 6}{-60x^2 - 240x - 180}$$

$$\frac{186}{186} \frac{186x + 186}{x + 1}$$

$$\frac{x + 1}{x^3 + x} \frac{x^2 + 4x + 3}{x + 3} (x + 3)$$

$$\frac{3x + 3}{3x + 3}$$

Dividing both expressions by $x + 1$, we obtain as L. C. M. $(x + 1)(x + 3)(x^4 - x^3 + 6x^2 - 6x + 6)$.

$$\begin{array}{r}
 25. \quad \frac{6x^3 - 10x^2 - 3x + 5}{6x^3 - 10x^2 - 3x + 5} \cdot \frac{6x^3 - 7x^2 - 2x - 5}{6x^3 - 10x^2 - 3x + 5} \\
 \frac{6x^3 + x - 10}{6x^3 + 2x^2 - 20x} \cdot \frac{6x^3 - 10x^2 - 3x + 5}{-12x^3 + 17x + 5} \\
 \frac{7) 21x - 35}{-12x^3 - 4x + 40} \\
 \frac{3x - 5}{3x^2 + x - 10(x + 2)} \\
 \frac{6x - 10}{6x - 10}
 \end{array}$$

Dividing both expressions by $3x - 5$, we obtain as l. c. m. $(3x - 5)(2x^3 - 1)(2x^2 + x + 1)$.

26. Divide one expression by 3, and the other by $2x$;

$$\begin{array}{r}
 \frac{x^3 - x - 2}{x^3 - x^2 - 2x} \cdot \frac{x^3 - 5x + 2}{x^3 - x^2 - 2x} \cdot \frac{(x + 1)}{x^3 - x^2 - 2x} \\
 \frac{-2}{x - 2} \cdot \frac{2x + 4}{x - 2} \cdot \frac{x^3 - x - 2}{x^3 - 2x} \\
 \frac{x - 2}{x^3 - 2x} \cdot \frac{x^3 - x - 2}{x^3 - 2x} \\
 \frac{x - 2}{x - 2}
 \end{array}$$

Dividing both expressions by $x - 2$, we obtain $x + 1$ and $x^2 + 2x - 1$; also $6x$ is l. c. m. of 3 and $2x$; hence the complete l. c. m. is $6x(x - 2)(x + 1)(x^2 + 2x - 1)$.

[illegible]

Dividing both expressions by $x - 1$, we obtain as L. C. M.

$$(x-1)(3x-2)(4x^2-1).$$

28. Divide the first expression by $2a$, and the second by 3: then

$$\begin{array}{r} a^3 - 5a + 6 \overline{) a^4 - 5a^3 + 8a^2 - 8(a^3 + 2)} \\ \underline{a^4 - 5a^3 + 6a^2} \\ 2a^2 - 8 \\ \underline{2a^2 - 10a + 12} \\ 10 \overline{) 10a - 20} \\ \underline{10a - 20} \\ a - 2 \overline{) a^3 - 5a + 6(a - 3)} \\ \underline{a^3 - 2a} \\ -3a + 6 \\ \underline{-3a + 6} \\ 0 \end{array}$$

Dividing both expressions by $a - 2$, we find as L. C. M.

$$6a(a-2)(a-3)(a^3-3a^2+2a+4).$$

$$\begin{array}{r} 29. \quad x^3 - x^2 + x + 8 \overline{) x^4 + x^3 - 3x^2 - x + 2(x+2)} \\ \underline{x^4 - x^3 + x^2 + 3x} \\ 2x^3 - 4x^2 - 4x + 2 \\ \underline{2x^3 - 2x^2 + 2x + 6} \\ -2 \underline{-2x^2 - 6x - 4} \\ x^2 + 3x + 2 \end{array}$$

Dividing the two expressions by $x+1$, we obtain as L. C. M.

$$(x+1)(x^2-2x+3)(x^3-3x+2).$$

31.

$$\begin{array}{r}
 x^3 + 6x + 5 \overline{) x^3 + 7x + 6} \quad (1 \\
 \underline{x^3 + 6x + 5} \\
 x + 1 \overline{) x^3 + 6x + 5} \quad (x + 5 \\
 \underline{x^2 + x} \\
 5x + 5 \\
 \underline{5x + 5} \\
 0
 \end{array}$$

Hence L. C. M. of the first two expressions is

$$(x+1)(x+5)(x+6) = x^3 + 12x^2 + 41x + 30.$$

Then to find L. C. M. of this result and the third expression, we have

$$\begin{array}{r}
 x^3 + 11x + 30 \overline{) x^3 + 12x^2 + 41x + 30} \quad (x + 1 \\
 \underline{x^3 + 11x^2 + 30x} \\
 x^2 + 11x + 30 \\
 \underline{x^2 + 11x + 30} \\
 0
 \end{array}$$

this shows that the third expression also divides into the L. C. M. without remainder; hence the L. C. M. of the three is $(x+1)(x+5)(x+6)$.

32.

$$\begin{array}{r}
 x^3 + 2x - 15 \overline{) x^3 - 5x + 6} \quad (1 \\
 \underline{x^3 + 2x - 15} \\
 -7 \overline{) -7x + 21} \\
 \underline{-7x - 3} \\
 x - 3 \overline{) x^3 + 2x - 15} \quad (x + 5 \\
 \underline{x^3 - 3x} \\
 5x - 15 \\
 \underline{5x - 15} \\
 0
 \end{array}$$

Hence L. C. M. of the first two expressions is $(x-3)(x+5)(x-2)$ or $x^3 - 19x + 30$; then taking this with the third expression, we have

$$\begin{array}{r}
 x^3 - 19x + 30 \overline{) x^3 + 5x^2 + x + 5} \quad (1 \\
 \underline{x^3 - 19x + 30} \\
 5 \overline{) 5x^2 + 20x - 25} \\
 \underline{5x^2 + 4x - 5} \\
 x^3 - 19x + 30 \quad (x - 4 \\
 \underline{x^3 + 4x^2 - 5x} \\
 -4x^2 - 14x + 30 \\
 \underline{-4x^2 - 16x + 20} \\
 2 \overline{) 2x + 10} \\
 \underline{2x + 5} \\
 x + 5 \overline{) x^3 + 4x - 5} \quad (x - 1 \\
 \underline{x^2 + 5x} \\
 -x - 5 \\
 \underline{-x - 5} \\
 0
 \end{array}$$

Hence the complete L. C. M. is $(x+5)(x-3)(x-2)(x^2+1)$.

33.

$$\begin{array}{r}
 x^3 - 4 \quad x^3 - 7x + 10 \quad (1 \\
 \quad \quad \quad x^2 \quad \quad \quad - 4 \\
 \hline
 - 7 \quad - 7x + 14 \\
 \quad \quad \quad x - 2 \quad x^2 - 4 \quad (x + 2 \\
 \quad \quad \quad \quad \quad x^2 - 2x \\
 \hline
 \quad \quad \quad \quad \quad 2x - 4 \\
 \quad \quad \quad \quad \quad 2x - 4 \\
 \hline
 \quad \quad \quad \quad \quad 0
 \end{array}$$

Hence L. C. M. of the first two is $(x-2)(x+2)(x-5)$ or $x^3 - 5x^2 - 4x + 20$.

Now take this with the third expression:

$$\begin{array}{r}
 x^3 - 5x^2 - 4x + 20 \quad x^3 - 5x^2 + 4x - 20 \quad (1 \\
 \quad \quad \quad x^3 - 5x^2 - 4x + 20 \\
 \hline
 \quad \quad \quad 8 \quad 8x - 40 \\
 \quad \quad \quad \quad \quad x - 5 \quad x^3 - 5x^2 - 4x + 20 \quad (x^2 - 4 \\
 \quad \quad \quad \quad \quad \quad \quad x^3 - 5x^2 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad - 4x + 20 \\
 \quad \quad \quad \quad \quad \quad \quad \quad - 4x + 20 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

Hence the complete L. C. M. is $(x-2)(x+2)(x-5)(x^2+4)$.

34.

$$\begin{array}{r}
 6x^2 - 7x - 20 \quad 9x^2 - 16 \\
 \quad \quad \quad \quad \quad 2 \\
 \hline
 18x^2 - 32 \quad (3 \\
 18x^2 - 21x - 60 \\
 \hline
 \quad \quad \quad 7 \quad 21x + 28 \\
 \quad \quad \quad \quad \quad 3x + 4 \quad 6x^2 - 7x - 20 \quad (2x - 5 \\
 \quad \quad \quad \quad \quad \quad \quad 6x^2 + 8x \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad - 15x - 20 \\
 \quad \quad \quad \quad \quad \quad \quad \quad - 15x - 20 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

Hence the L. C. M. of the first two expressions is

$$(3x+4)(2x-5)(3x-4), \text{ or } 18x^3 - 45x^2 - 32x + 80.$$

Now take this with the third expression:

$$\begin{array}{r}
 6x^3 - 13x^2 - 9x + 10 \quad 18x^3 - 45x^2 - 32x + 80 \quad (3 \\
 \quad \quad \quad 18x^3 - 39x^2 - 27x + 30 \\
 \hline
 \quad \quad \quad - 6x^2 - 5x + 50 \quad 6x^3 - 13x^2 - 9x + 10 \quad (-x + 3 \\
 \quad \quad \quad \quad \quad \quad \quad 6x^3 + 5x^2 - 50x \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad - 18x^2 + 41x + 10 \\
 \quad \quad \quad \quad \quad \quad \quad \quad - 18x^2 - 15x + 150 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad 28 \quad 56x - 140 \\
 \quad \quad \quad \quad \quad \quad \quad \quad 2x - 5 \quad 6x^2 + 5x - 50 \quad (3x \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 6x^2 - 15x \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 20x - 50 \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 20x - 50 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

Hence the complete L. C. M. is $(2x-5)(3x+4)(3x-4)(3x^2+x-2)$.

$$\begin{array}{r}
 35. \quad 2x^3 - x^2 + 6x - 8 \quad 2x^3 - 8x^2 - x + 1 \quad (1) \\
 \underline{2x^3 - x^2 + 6x - 8} \\
 -2x^3 - 7x + 4 \quad 2x^3 - x^2 + 6x - 8 \quad (-x + 4) \\
 \underline{2x^3 + 7x^2 - 4x} \\
 -8x^2 + 10x - 8 \\
 \underline{-8x^2 - 28x + 16} \\
 19 \quad 38x - 19 \\
 \underline{2x - 1} \quad 2x^2 + 7x - 4 \quad (x + 4) \\
 \underline{2x^2 - x} \\
 8x - 4 \\
 \underline{8x - 4}
 \end{array}$$

Hence L. C. M. of the first two expressions is

$$(2x - 1)(x^2 + 8)(x^2 - x - 1) = 2x^5 - 3x^4 + 5x^3 - 8x^2 - 3x + 8.$$

Now take this and the other expression ;

$$\begin{array}{r}
 x^4 - x^3 + 2x^2 - 3x - 3 \quad 2x^5 - 3x^4 + 5x^3 - 8x^2 - 3x + 8 \quad (2x - 1) \\
 \underline{2x^5 - 2x^4 + 4x^3 - 6x^2 - 6x} \\
 -x^4 + x^3 - 2x^2 + 3x + 3 \\
 \underline{-x^4 + x^3 - 2x^2 + 3x + 3}
 \end{array}$$

this shews that the third expression is contained in the L. C. M. already obtained.

EXERCISE XVI.

1. Multiply (1) by 2; then $2x + 4y + 6z = 40$;
 and from (2) $2x + 3y + 2z = 18$;
 \therefore by subtraction $y + 4z = 22$(A).
 Again multiply (1) by 5; then $5x + 10y + 15z = 100$;
 and from (3) $5x + 2y + 4z = 29$;
 \therefore by subtraction $8y + 11z = 71$(B).
 Now take together (A) and (B);
 multiply (A) by 8, then $8y + 32z = 176$,
 and from (B) $8y + 11z = 71$;
 \therefore by subtraction $21z = 105$, or $z = 5$;
 substitute this value of z in (A); then
 $y + 20 = 22$, or $y = 2$;
 substitute values of y and z in (1); then
 $x + 4 + 15 = 20$, or $x = 1$.

2. From (1) $2x + 5y - 2z = 4$,
 from (2) $3x + 7y + 2z = 35$;
 \therefore by addition $5x + 12y = 39$ (A).
 Again multiply (2) by 5; then $15x + 35y + 10z = 175$,
 and multiply (3) by 2; $4x + 4y + 10z = 80$;
 \therefore by subtraction $11x + 31y = 95$ (B).

Take together (A) and (B);
 from (A) $55x + 132y = 429$,
 from (B) $55x + 155y = 475$,
 \therefore by subtraction $-23y = -46$, or $y = 2$;
 substitute in (A), then $5x + 24 = 39$, or $x = 8$;
 substitute in (1), then $6 + 10 - 2z = 4$, or $z = 6$.

3. Multiply (1) by 3; then $9x + 3y + 21z = 72$,
 and from (2) $2x + 3y + 4z = 27$;
 \therefore by subtraction $7x + 17z = 45$ (A),
 again, multiply (1) by 4; then $12x + 4y + 28z = 96$,
 and from (3) $5x + 4y - 16z = 24$,
 \therefore by subtraction $7x + 44z = 72$ (B).

Now subtract (A) from (B); thus $27z = 27$, or $z = 1$;
 substitute this value of z in A $7x + 17 = 45$, or $x = 4$;
 and then from (1), $12 + y + 7 = 24$, or $y = 5$.

4. Multiply (2) by 3; then $9x - 12y + 3z = 57$,
 and from (1) $4x + 6y + 8z = 11$,
 \therefore by subtraction $5x - 18y = 46$ (A).
 Again, multiply (2) by 8; then $24x - 32y + 8z = 152$,
 and from (3) $7x + 5y + 8z = 44$,
 \therefore by subtraction $17x - 37y = 108$ (B).

Multiply (A) by 17, and (B) by 5; then
 $85x - 306y = 782$;
 $85x - 185y = 540$;
 $\therefore -121y = 242$, or $y = -2$;
 substitute in (A), then $5x + 36 = 46$, or $x = 2$;
 and then from (2), $6 + 8 + z = 19$, or $z = 5$.

5. Multiply (1) by 2; then $10x + 4y - 10z = 46$,
 and from (2) $10x + 3y + 4z = 0$,
 \therefore by subtraction $y - 14z = 46$ (A).
 Again multiply (3) by 5; then $10x - 25y - 85z = 5$,
 and from (2) $10x + 3y + 4z = 0$,
 \therefore by subtraction $-28y - 89z = 5$ (B).

Take together (A) and (B); thus

$$\begin{array}{rcl}
 & 28y - 89z = 1288, \\
 & -28y - 89z = 5, \\
 \therefore \text{by addition} & & -431z = 1293, \text{ or } z = -3; \\
 \text{substitute in (A), then} & & y + 42 = 46, \text{ or } y = 4: \\
 \text{substitute in (1), then} & & 5x + 8 + 15 = 23, \text{ or } x = 0.
 \end{array}$$

$$\begin{array}{rcl}
 6. \text{ Multiply (2) by 3, then} & 12x + 9y - 15z = 15, \\
 \text{and from (1),} & 3x + 9y - 7z = 7, \\
 \therefore \text{by subtraction} & & 9x - 8z = 8 \dots\dots\dots (A). \\
 \text{Again multiply (3) by 3, then} & 30x - 3y - 3z = 3, \\
 \text{and from (2)} & & 4x + 3y - 5z = 5, \\
 \therefore \text{by addition} & & 34x - 8z = 8 \dots\dots\dots (B).
 \end{array}$$

$$\begin{array}{rcl}
 \text{Now subtract (A) from (B), then} & & 25x = 0, \text{ or } x = 0: \\
 \text{substitute in (A), then} & & -8z = 8, \text{ or } z = -1: \\
 \text{substitute in (1), then} & & 9y + 7 = 7, \text{ or } y = 0.
 \end{array}$$

$$\begin{array}{rcl}
 7. \text{ Multiply (1) by 3; then} & 15x - 9y + 3z = 15, \\
 \text{and from (2)} & & 2x + y + 3z = 19, \\
 \therefore \text{by subtraction} & & 13x - 10y = -4 \dots\dots\dots (A). \\
 \text{Again add together (1) and (3), then} & & 6x - y = 9 \dots\dots\dots (B).
 \end{array}$$

$$\begin{array}{rcl}
 \text{Now take together (A) and (B); thus} & & 13x - 10y = -4, \\
 & & 60x - 10y = 90, \\
 \therefore \text{by subtraction} & & -47x = -94, \text{ or } x = 2: \\
 \text{substitute in (B), then} & & 12 - y = 9, \text{ or } y = 3: \\
 \text{substitute in (1), then} & & 10 - 9 + z = 5, \text{ or } z = 4.
 \end{array}$$

$$\begin{array}{rcl}
 8. \text{ Multiply (1) by 4, and (2) by 3; then} & & 12x + 16y - 20z = 12, \\
 & & 12x - 15y + 9z = 39, \\
 \therefore \text{by subtraction} & & 31y - 29z = -27 \dots\dots\dots (A). \\
 \text{Again multiply (1) by 5, and (3) by 3; then} & & 15x + 20y - 25z = 15, \\
 & & 15x + 9y + 12z = 78, \\
 \therefore \text{by subtraction} & & 11y - 37z = -63 \dots\dots\dots (B).
 \end{array}$$

$$\begin{array}{rcl}
 \text{Now take together (A) and (B), thus} & & 341y - 319z = -297, \\
 & & 341y - 1147z = -1953, \\
 \therefore \text{by subtraction} & & 828z = 1656, \text{ or } z = 2: \\
 \text{substitute in (B), then} & & 11y - 74 = -63, \text{ or } y = 1: \\
 \text{substitute in (1), then} & & 3x + 4 - 10 = 3, \text{ or } x = 3.
 \end{array}$$

9. Multiply (3) by 2, then $8x + 2y + 6z = 16$,
 and from (1) $3x + 2y - 7z = 19$,
 \therefore by subtraction, $5x + 13z = -3$ (A).

Again multiply (3) by 3, then $12x + 3y + 9z = 24$,
 and from (2) $2x - 3y - 5z = 0$,
 \therefore by addition, $14x + 4z = 24$ (B).

Now take together (A) and (B), thus

$$\begin{aligned} 70x + 182z &= -42, \\ 70x + 20z &= 120, \\ \therefore 162z &= -162, \text{ or } z = -1; \end{aligned}$$

substitute in (A), then

$$5x - 13 = -3, \text{ or } x = 2:$$

substitute in (1), then

$$6 + 2y + 7 = 19, \text{ or } y = 8.$$

10. Multiply (1) by 3, then $33x + 3y + 3z = 48$,
 and from (2) $2x + 3y + 4z = 20$,
 \therefore by subtraction $31x - z = 28$ (A).

Again multiply (2) by 2, then $4x + 6y + 8z = 40$,
 and from (3), $5x + 6y + 7z = 38$,
 \therefore by subtraction $-x + z = 2$ (B).

Add together (A) and (B), then $30x = 30$, or $x = 1$:

substitute in (B), then

$$-1 + z = 2, \text{ or } z = 3:$$

substitute in (1), then

$$11 + y + 3 = 16, \text{ or } y = 2.$$

11. Multiply (2) by 3, then $9x - 3y + 6z = 24$,
 and from (1) $5x + 3y - 6z = 4$,
 \therefore by addition $14x = 28$, or $x = 2$ (A).

Again by subtracting (3) from (2), $2x + y = 6$,
 and substituting from (A), $4 + y = 6$, or $y = 2$:
 substitute in (3), then $2 - 4 + 2z = 2$, or $z = 2$.

12. Multiply (1) by 3, then $12x - 9y + 6z = 120$,
 and from (2) $5x + 9y - 7z = 47$,
 \therefore by addition $17x - z = 167$ (A).

Again, multiply (1) by 8, then $32x - 24y + 16z = 320$,
 and multiply (3) by 3, then $27x + 24y - 9z = 291$,
 \therefore by addition, $59x + 7z = 611$ (B).

Now multiply (A) by 7, then $119x - 7z = 1169$,

\therefore by addition

$$178x = 1780, \text{ or } x = 10:$$

substitute in (A), then

$$170 - z = 167, \text{ or } z = 3:$$

substitute in (1), then

$$40 - 8y + 6 = 40, \text{ or } y = 2.$$

13. Multiply (2) by 2, then $8x + 6z = 40$,
 and from (3) $8y + 6z = 27$,
 \therefore by subtraction $8x - 8y = 13$ (A);
 but from (1) $3x + 2y = 8$ (1).

Now multiply (A) by 2, then $16x - 6y = 26$,
 and multiply (1) by 3, then $9x + 6y = 24$,
 \therefore by addition $25x = 50$, or $x = 2$:
 substitute in (1), then $6 + 2y = 8$, or $y = 1$:
 substitute in (2), then $8 + 3z = 20$, or $z = 4$.

14. Take together (2) and (3), thus
 $10x - 4z = 10$,
 $3y + 4z = 20$,
 \therefore by addition $10x + 3y = 30$ (A);
 but from (1) $4x + 7y = 12$ (1);
 hence $20x + 6y = 60$,
 and $20x + 35y = 60$,
 \therefore by subtraction $-29y = 0$, or $y = 0$:
 substitute in (1), then $4x = 12$, or $x = 3$:
 substitute in (3), then $4z = 20$, or $z = 5$.

15. Take together (1) and (3), thus
 $4x + 14y = 30$,
 $4x + 9z = 23$,
 \therefore by subtraction $14y - 9z = 7$ (A).
 Now multiply (2) by 3, $6y + 9z = 33$,
 \therefore by addition $20y = 40$, or $y = 2$:
 substitute in (1), then $2x + 14 = 15$, or $x = \frac{1}{2}$:
 substitute in (3), then $2 + 9z = 23$, or $z = 2\frac{1}{9}$.

16. Take together (2) and (3), thus
 $10x + 5y + 5z = 165$,
 $9y - 5z = 34$,
 \therefore by addition $10x + 14y = 199$ (A).
 Now take together (A) and (1), thus
 $70x + 98y = 1398$,
 $70x - 15y = 150$,
 \therefore by subtraction, $113y = 1248$, or $y = 11$:
 substitute in (1), then $14x - 33 = 30$, or $x = 4\frac{1}{2}$:
 substitute in (2), then $99 - 5z = 34$, or $z = 13$.

17. From (1),
from (2),
∴ by subtraction
Now from (3)
and from (4)
∴ by addition,
substitute in (2),
substitute in (3),

$$\begin{aligned} 14x - 4y &= 40, \\ 14x - 21z &= 28, \\ -4y + 21z &= 12 \dots\dots\dots (4). \\ 12y - 16z &= -36, \\ -12y + 63z &= 36; \\ 47z &= 0, \text{ or } z = 0: \\ 2x &= 4, \text{ or } x = 2: \\ 8y &= -9, \text{ or } y = -3. \end{aligned}$$

18. From (3),
from (1),
∴ by subtraction,
Again, from (2)
and from (4)
∴ by subtraction,
substitute in (2),
substitute in (3),

$$\begin{aligned} 8x - 6z &= 24, \\ 3x - 5y &= 17, \\ 5y - 6z &= 7 \dots\dots\dots (4). \\ 10y - 25z &= 40, \\ 10y - 12z &= 14, \\ -13z &= 26, \text{ or } z = -2: \\ 2y + 10 &= 8, \text{ or } y = -1: \\ x + 4 &= 8, \text{ or } x = 4. \end{aligned}$$

19. From (3)
from (1)
∴ by subtraction
Now from (2)
and from (4)
∴ by subtraction,
substitute in (2),
substitute in (1),

$$\begin{aligned} 2x + 8y - 4z &= 30, \\ 2x - 3y &= 8, \\ 11y - 4z &= 22 \dots\dots\dots (4). \\ 55y - 99z &= 110, \\ 55y - 20z &= 110, \\ -79z &= 0, \text{ or } z = 0: \\ 5y &= 10, \text{ or } y = 2: \\ 2x - 6 &= 8, \text{ or } x = 7. \end{aligned}$$

20. From (2),
from (3),
∴ by subtraction,
Again, from (1)
from (4)
∴ by subtraction
substitute in (1), then
substitute in (2), then

$$\begin{aligned} 3x + 12y + 6z &= 27, \\ 12x + 12y + 16z &= 124, \\ 9x + 10z &= 97 \dots\dots\dots (4). \\ 18x + 27z &= 243, \\ 18x + 20z &= 194, \\ 7z &= 49, \text{ or } z = 7: \\ 2x + 21 &= 27, \text{ or } x = 3: \\ 3 + 4y + 14 &= 9, \text{ or } y = -2. \end{aligned}$$

21. From (1), (L. C. D. 12) $4x + 3y + 6z = 24$(1),
from (2), (L. C. D. 30) $5x + 30y + 3z = 60$(2),
from (3), (L. C. D. 4) $4x + 4y + z = 1$(3).

Multiply (2) by 2, and subtract (1) from result;
then
or, dividing by 3,

$$\begin{aligned} 6x + 57y &= 96, \\ 2x + 19y &= 32 \dots\dots\dots (4). \end{aligned}$$

Again multiply (3) by 3, and subtract (2) from it;
then

$$7x - 18y = -57 \dots\dots\dots (B).$$

From (A)

$$14x + 138y = 224,$$

from (B)

$$14x - 36y = -114;$$

\therefore by subtraction

$$169y = 338, \text{ or } y = 2:$$

substitute in (A), then

$$2x + 38 = 32, \text{ or } x = -3:$$

substitute in (3), then

$$-12 + 8 + z = 1, \text{ or } z = 5.$$

22. From (1), (L. C. D. 10) $5x + 10y + 2z = 10 \dots\dots\dots (1),$

from (2), (L. C. D. 40) $10x - 5y - 4z = 80 \dots\dots\dots (2).$

Multiply (2) by 2, and add to (1), then

$$25x - 6z = 170 \dots\dots\dots (A).$$

Now from (3),

$$9x - 12z = 12,$$

and from (A)

$$50x - 12z = 340,$$

$$\therefore 41x = 328, \text{ or } x = 8:$$

substitute in (3),

$$24 - 4z = 4, \text{ or } z = 5:$$

substitute in (1),

$$40 + 10y + 10 = 10, \text{ or } y = -4.$$

23. From (1), (L. C. D. 15) $3x - 5y = 0 \dots\dots\dots (1),$

from (2), (L. C. D. 10) $5x + 2z = 50 \dots\dots\dots (2).$

Multiply (3) by 2, and subtract from (2),

hence $8x + 2y = 42 \dots\dots\dots (A).$

Now subtract (1) from (A), then $7y = 42, \text{ or } y = 6:$

substitute in (1), $3x - 30 = 0, \text{ or } x = 10:$

substitute in (3), $10 - 6 + z = 4, \text{ or } z = 0.$

24. From (1), (L. C. D. 12) $3x = 36 - 4z \dots\dots\dots (1),$

from (2), (L. C. D. 40) $4x = 40 - 5y \dots\dots\dots (2),$

or, changing the order of letters, $3x + 4z = 36 \dots\dots\dots (1),$

$$4x + 5y = 40 \dots\dots\dots (2),$$

$$y - z = -2 \dots\dots\dots (3).$$

Multiply (3) by 4, and add to (1), then

$$3x + 4y = 28 \dots\dots\dots (A).$$

Take together (2) and (A); then

from (2), $12x + 15y = 120,$

$$12x + 16y = 112,$$

\therefore by subtraction,

$$y = -8:$$

substitute in (2),

$$4x - 40 = 40; \text{ or } x = 20:$$

substitute in (3),

$$-8 - z = -2; \text{ or } z = -6.$$

25. From (1), $2y + z = 3$ (1),
 from (2), $x + 5y - z = 0$ (2),
 from (3), $x - 2z = 0$ (3).
 Subtract (3) from (2), then
 $5y + z = 0$ (4);
 subtract (1) from (4), then
 $3y = -3$, or $y = -1$;
 substitute in (1),
 $-2 + z = 3$, or $z = 5$;
 substitute in (3),
 $x - 10 = 0$, or $x = 10$.

26. From (1), (L. C. D. 60), $12x + 20y - 15z = 0$ (1),
 from (2), (L. C. D. 60) $6x - 5y = 15$ (2),
 from (3), $x + y - z = 0$ (3).

Multiply (3) by 15, $15x + 15y - 15z = 0$;
 \therefore by subtraction (1), $3x - 5y = 0$ (4).

Multiply (4) by 2, $6x - 10y = 0$;
 \therefore by subtraction from (2),
 $5y = 15$, or $y = 3$;
 substitute in (4),
 $3x - 15 = 0$, or $x = 5$;
 substitute in (3),
 $5 + 3 - z = 0$, or $z = 8$.

27. From (1), $2x + z = 0$ (1),
 from (2), $6x - 7y + 6z = 0$ (2),
 from (3), $3x - 2y + 2z = -5$ (3);
 take together (2) and (3), $12x - 14y + 12z = 0$,
 $21x - 14y + 14z = -35$,
 \therefore by subtraction $9x + 2z = -35$ (4).

Multiply (1) by 2, $4x + 2z = 0$,
 \therefore by subtraction $5x = -35$, or $x = -7$;
 substitute in (1), then
 $-14 + z = 0$, or $z = 14$;
 substitute in (3), then
 $-21 - 2y + 28 = -5$, or $y = 6$.

28. $x + 2z = 13$ (1),
 $4x + 2y = 12$ (2),
 $5y + 3z = 15$ (3).

Multiply (1) by 4, and subtract from (2),
 $\therefore 2y - 8z = -40$ (4).

Take together (4) and (3),
 $10y + 6z = 30$,
 $10y - 40z = -200$,
 $\therefore 46z = 230$, or $z = 5$;

substitute in (1), then
 $x + 10 = 13$, or $x = 3$;
 substitute in (2), then
 $12 + 2y = 12$, or $y = 0$.

29. From (1), $3x + y + z = 2z + 4$,
 or $3x + y - z = 4$ (1),
 from (2), $x + 2y = 2$ (2),
 from (3), $2x - 3y + 2z = 3z + 6$,
 or $2x - 3y - z = 4$ (3).
 Subtract (3) from (1), then $x + 4y = 0$ (A),
 subtract (2) from (A), then $2y = -2$, or $y = -1$:
 substitute in (2), $x - 2 = 2$, or $x = 4$:
 substitute in (1), $12 - 1 - z = 4$, or $z = 7$.
30. From (1), $x + 4y + 2z = 0$ (1),
 from (2), $24y - 16z = x$,
 or $-x + 24y - 16z = 0$ (2),
 from (3), $3x = 8x - 8y + 12z - 24$,
 or $-5x + 8y - 12z = -24$ (3).
 Now add together (1) and (2), then $28y - 14z = 0$ (A).
 Again, multiply (1) by 5, and add to (3), then $28y - 2z = -24$ (B);
 subtract (B) from (A), then $-12z = 24$, or $z = -2$:
 substitute in (A), then $28y + 28 = 0$, or $y = -1$:
 substitute in (1), then $x - 4 - 4 = 0$, or $x = 8$.

EXERCISE XVII.

1. $x = \pm 9$.
2. $x = \pm 12$.
3. $x^2 = 121$, $\therefore x = \pm 11$.
4. $x^2 = 9$, $\therefore x = \pm 3$.
5. $x^2 = 36$, $\therefore x = \pm 6$.
6. $x^2 = 25$, $\therefore x = \pm 5$.
7. $9x^2 - 2x^2 = 252$, $\therefore 7x^2 = 252$, $\therefore x^2 = 36$, $\therefore x = \pm 6$.
8. $4x^2 - 8x + 6x^2 - 9x = 10 - 5x - 12x$;
 $10x^2 = 10$, $\therefore x^2 = 1$, or $x = \pm 1$.
9. $5 - 2x - 2 + 2x - 2x^2 + 5 = 0$;
 $-2x^2 = -8$, $\therefore x^2 = 4$, or $x = \pm 2$.
10. $14x^2 + 10x - x^2 - x = 9x$;
 $13x^2 = 0$, $\therefore x = 0$.
11. $x^2 + 6x + 9 = 16 + 9 = 25$;
 $x + 3 = 5$, or -5 ;
 $x = 5 - 3$, or $-5 - 3$, i. e. 2 or -8 .

12. $x^2 + 10x + 25 = 39 + 25 = 64$;
 $x + 5 = 8$, or -8 ;
 $x = 8 - 5$, or $-8 - 5$, i. e. 3 or -13 .
13. $x^2 + 14x + 49 = 15 + 49 = 64$;
 $x + 7 = 8$, or -8 ;
 $x = 8 - 7$, or $-8 - 7$, i. e. 1 or -15 .
14. $x^2 + 20x + 100 = 96 + 100 = 196$;
 $x + 10 = 14$, or -14 ;
 $x = 14 - 10$, or $-14 - 10$, i. e. 4 or -24 .
15. $x^2 + 4x + 4 = 32 + 4 = 36$;
 $x + 2 = 6$, or -6 ;
 $x = 6 - 2$, or $-6 - 2$, i. e. 4 or -8 .
16. $x^2 + 4x + 4 = 140 + 4 = 144$;
 $x + 2 = 12$, or -12 ;
 $x = 12 - 2$, or $-12 - 2$, i. e. 10 or -14 .
17. $x^2 + 2x + 1 = 15 + 1 = 16$;
 $x + 1 = 4$, or -4 ;
 $x = 4 - 1$, or $-4 - 1$, i. e. 3 or -5 .
18. $x^2 + 2x + 1 = 63 + 1 = 64$;
 $x + 1 = \pm 8$;
 $x = 8 - 1$, or $-8 - 1$, i. e. 7 or -9 .
19. $x^2 + 48x + 576 = 580 + 576 = 1156$;
 $x + 24 = \pm 34$;
 $x = 34 - 24$, or $-34 - 24$, i. e. 10 or -58 .
20. $x^2 + 100x + 2500 = 4900$;
 $x + 50 = \pm 70$;
 $x = 70 - 50$, or $-70 - 50$, i. e. 20 or -120 .
21. $x^2 - 12x + 36 = 64$;
 $x - 6 = \pm 8$;
 $x = 8 + 6$, or $-8 + 6$, i. e. 14 or -2 .
22. $x^2 - 6x + 9 = 36$;
 $x - 3 = \pm 6$;
 $x = 6 + 3$, or $-6 + 3$, i. e. 9 or -3 .
23. $x^2 - 2x + 1 = 25$;
 $x - 1 = \pm 5$;
 $x = 5 + 1$, or $-5 + 1$, i. e. 6 or -4 .

24. $x^2=2x$, $\therefore x=2$ or 0 .
25. $x^2-6x+9=4$;
 $x-3=\pm 2$;
 $x=2+3$, or $-2+3$, i.e. 5 or 1 .
26. $x^2-8x+16=4$;
 $x-4=\pm 2$;
 $x=2+4$, or $-2+4$, i.e. 6 or 2 .
27. $x^2-20x+100=64$;
 $x-10=\pm 8$;
 $x=8+10$, or $-8+10$, i.e. 18 or 2 .
28. $x^2-84x+1764=1521$;
 $x-42=\pm 39$;
 $x=39+42$, or $-39+42$, i.e. 81 or 3 .
29. $x^2+96x+2304=2209$;
 $x+48=\pm 47$;
 $x=47-48$, or $-47-48$, i.e. -1 or -95 .
30. $x^2+42x+441=121$;
 $x+21=\pm 11$;
 $x=11-21$, or $-11-21$, i.e. -10 or -32 .
31. $x^2+7x+\frac{49}{4}=18+\frac{49}{4}=\frac{121}{4}$;
 $x+\frac{7}{2}=\pm\frac{11}{2}$;
 $x=\frac{11}{2}-\frac{7}{2}$, or $-\frac{11}{2}-\frac{7}{2}$, i.e. 2 or -9 .
32. $x^2+3x+\frac{9}{4}=10+\frac{9}{4}=\frac{49}{4}$;
 $x+\frac{3}{2}=\pm\frac{7}{2}$;
 $x=\frac{7}{2}-\frac{3}{2}$, or $-\frac{7}{2}-\frac{3}{2}$, i.e. 2 or -5 .
33. $x^2+5x+\frac{25}{4}=24+\frac{25}{4}=\frac{121}{4}$;
 $x+\frac{5}{2}=\pm\frac{11}{2}$;
 $x=\frac{11}{2}-\frac{5}{2}$, or $-\frac{11}{2}-\frac{5}{2}$, i.e. 3 or -8 .

EXERCISE XVII.

34. $x^2 + 9x + \frac{81}{4} = 70 + \frac{81}{4} = \frac{361}{4};$
 $x + \frac{9}{2} = \pm \frac{19}{2};$
 $x = \frac{19}{2} - \frac{9}{2}, \text{ or } -\frac{19}{2} - \frac{9}{2}, \text{ i.e. } 5 \text{ or } -14.$
35. $x^2 - 11x + \frac{121}{4} = 26 + \frac{121}{4} = \frac{225}{4};$
 $x - \frac{11}{2} = \pm \frac{15}{2};$
 $x = \frac{15}{2} + \frac{11}{2}, \text{ or } -\frac{15}{2} + \frac{11}{2}, \text{ i.e. } 13 \text{ or } -2.$
36. $x^2 - 21x + \frac{441}{4} = 100 + \frac{441}{4} = \frac{841}{4};$
 $x - \frac{21}{2} = \pm \frac{29}{2};$
 $x = \frac{29}{2} + \frac{21}{2}, \text{ or } -\frac{29}{2} + \frac{21}{2}, \text{ i.e. } 25 \text{ or } -4.$
37. $x^2 + x + \frac{1}{4} = 20 + \frac{1}{4} = \frac{81}{4};$
 $x + \frac{1}{2} = \pm \frac{9}{2};$
 $x = \pm \frac{9}{2} - \frac{1}{2} = 4 \text{ or } -5.$
38. $x^2 + x + \frac{1}{4} = 156 + \frac{1}{4} = \frac{625}{4};$
 $x + \frac{1}{2} = \pm \frac{25}{2};$
 $x = \pm \frac{25}{2} - \frac{1}{2} = 12 \text{ or } -13.$
39. $x^2 - 13x + \frac{169}{4} = -22 + \frac{169}{4} = \frac{81}{4};$
 $x - \frac{13}{2} = \pm \frac{9}{2};$
 $x = \pm \frac{9}{2} + \frac{13}{2} = 11 \text{ or } 2.$

EXERCISE XVII.

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40.

$$x^2 - 58x + \frac{2809}{4} = -150 + \frac{2809}{4} = \frac{2209}{4};$$

$$x - \frac{58}{2} = \pm \frac{47}{2};$$

$$x = \pm \frac{47}{2} + \frac{58}{2} = 50 \text{ or } 3.$$

41.

$$x^2 + 2x = 8;$$

$$x^2 + 2x + 1 = 9;$$

$$x + 1 = \pm 3;$$

$$x = 3 - 1, \text{ or } -3 - 1, \text{ i.e. } 2 \text{ or } -4.$$

42.

$$x^2 + 8x = 4;$$

$$x^2 + 8x + \frac{9}{4} = 4 + \frac{9}{4} = \frac{25}{4};$$

$$x + \frac{3}{2} = \pm \frac{5}{2};$$

$$x = \pm \frac{5}{2} - \frac{3}{2} = 1 \text{ or } -4.$$

43.

$$x^2 + 4x = 5;$$

$$x^2 + 4x + 4 = 9;$$

$$x + 2 = \pm 3;$$

$$x = \pm 3 - 2 = 1 \text{ or } -5.$$

44.

$$x^2 + 21x = 100;$$

$$x^2 + 21x + \frac{441}{4} = 100 + \frac{441}{4} = \frac{841}{4};$$

$$x + \frac{21}{2} = \pm \frac{29}{2};$$

$$x = \pm \frac{29}{2} - \frac{21}{2} = 4 \text{ or } -25.$$

45.

$$x^2 - x = 2;$$

$$x^2 - x + \frac{1}{4} = 2 + \frac{1}{4} = \frac{9}{4};$$

$$x - \frac{1}{2} = \pm \frac{3}{2};$$

$$x = \pm \frac{3}{2} + \frac{1}{2} = 2 \text{ or } -1.$$

46. $x^2 + 6x + 9 = 16 + 9 = 25;$
 $x + 3 = \pm 5;$
 $x = \pm 5 - 3 = 2 \text{ or } -8.$
47. $x^2 + 10x + 25 = 100;$
 $x + 5 = \pm 10;$
 $x = \pm 10 - 5 = 5 \text{ or } -15.$
48. $x^2 + 9x + \frac{81}{4} = 22 + \frac{81}{4} = \frac{169}{4};$
 $x + \frac{9}{2} = \pm \frac{13}{2};$
 $x = \pm \frac{13}{2} - \frac{9}{2} = \frac{4}{2} \text{ or } -\frac{22}{2} = 2 \text{ or } -11.$
49. $x^2 - 7x + \frac{49}{4} - 12 + \frac{49}{4} = \frac{1}{4};$
 $x - \frac{7}{2} = \pm \frac{1}{2};$
 $x = \pm \frac{1}{2} + \frac{7}{2} = \frac{8}{2} \text{ or } \frac{6}{2} = 4 \text{ or } 3.$
50. $x^2 - 5x + \frac{25}{4} = 66 + \frac{25}{4} = \frac{269}{4};$
 $x - \frac{5}{2} = \pm \frac{17}{2};$
 $x = \pm \frac{17}{2} + \frac{5}{2} = 11 \text{ or } -5.$
51. $x^2 + \frac{3x}{2} + \frac{9}{16} = 7 + \frac{9}{16} = \frac{121}{16};$
 $x + \frac{3}{4} = \pm \frac{11}{4};$
 $x = \pm \frac{11}{4} - \frac{3}{4} = 2 \text{ or } -3\frac{1}{2}.$
52. $x + \frac{2x}{3} + \frac{1}{9} = 87 + \frac{1}{9} = \frac{784}{9};$
 $x + \frac{1}{3} = \pm \frac{28}{3};$
 $x = \pm \frac{28}{3} - \frac{1}{3} = 9 \text{ or } -9\frac{1}{3}.$

53. $x^2 + \frac{7x}{4} + \frac{49}{64} = 23 + \frac{49}{64} = \frac{1521}{64};$
 $x + \frac{7}{8} = \pm \frac{39}{8};$
 $x = \pm \frac{39}{8} - \frac{7}{8} = 4 \text{ or } -5\frac{1}{4}.$
54. $x^2 + \frac{2x}{5} + \frac{1}{25} = 51\frac{1}{5} + \frac{1}{25} = 51\frac{1}{5};$
 $x + \frac{1}{5} = \pm 7\frac{1}{5};$
 $x = \pm 7\frac{1}{5} - \frac{1}{5} = 7 \text{ or } -7\frac{1}{5}.$
55. $x^2 + \frac{3x}{10} + \frac{9}{400} = 17\frac{1}{5} + \frac{9}{400} = \frac{6889}{400};$
 $x + \frac{3}{20} = \pm \frac{83}{20};$
 $x = \pm \frac{83}{20} - \frac{3}{20} = 4 \text{ or } -4\frac{1}{5}.$
56. $x^2 - \frac{x}{4} + \frac{1}{64} = 84\frac{1}{4} + \frac{1}{64} = \frac{2209}{64};$
 $x - \frac{1}{8} = \pm \frac{47}{8};$
 $x = \pm \frac{47}{8} + \frac{1}{8} = 6 \text{ or } -5\frac{1}{4}.$
57. $x^2 + \frac{x}{3} = \frac{14}{9};$
 $x^2 + \frac{x}{3} + \frac{1}{36} = \frac{14}{9} + \frac{1}{36} = \frac{169}{36};$
 $x + \frac{1}{6} = \pm \frac{13}{6};$
 $x = \pm \frac{13}{6} - \frac{1}{6} = 2 \text{ or } -2\frac{1}{3}.$
58. $x^2 + \frac{3x}{5} = \frac{92}{5};$
 $x^2 + \frac{3x}{5} + \frac{9}{100} = \frac{92}{5} + \frac{9}{100} = \frac{1849}{100};$
 $x + \frac{3}{10} = \pm \frac{43}{10};$
 $x = \pm \frac{43}{10} - \frac{3}{10} = 4 \text{ or } -4\frac{1}{5}.$

59.

$$x^2 + \frac{7x}{12} = \frac{31}{6};$$

$$x^2 + \frac{7x}{12} + \frac{49}{576} = \frac{31}{6} + \frac{49}{576} = \frac{2976 + 49}{576} = \frac{3025}{576};$$

$$x + \frac{7}{24} = \pm \frac{55}{24};$$

$$x = \pm \frac{55}{24} - \frac{7}{24} = 2 \text{ or } -2\frac{1}{3}.$$

60.

$$x^2 - \frac{2x}{3} = \frac{8}{3};$$

$$x^2 - \frac{2x}{3} + \frac{1}{9} = \frac{8}{3} + \frac{1}{9} = \frac{25}{9};$$

$$x - \frac{1}{3} = \pm \frac{5}{3};$$

$$x = \pm \frac{5}{3} + \frac{1}{3} = 2 \text{ or } -1\frac{1}{3}.$$

61.

$$x^2 - 5x = -6;$$

$$x^2 - 5x + \frac{25}{4} = -6 + \frac{25}{4} = \frac{1}{4};$$

$$x - \frac{5}{2} = \pm \frac{1}{2};$$

$$x = \pm \frac{1}{2} + \frac{5}{2} = 3 \text{ or } 2.$$

62.

$$x^2 - 3x = 4;$$

$$x^2 - 3x + \frac{9}{4} = 4 + \frac{9}{4} = \frac{25}{4};$$

$$x - \frac{3}{2} = \pm \frac{5}{2};$$

$$x = \pm \frac{5}{2} + \frac{3}{2} = 4 \text{ or } -1.$$

63.

$$x^2 + 7x = 80;$$

$$x^2 + 7x + \frac{49}{4} = 80 + \frac{49}{4} = \frac{169}{4};$$

$$x + \frac{7}{2} = \pm \frac{13}{2};$$

$$x = \pm \frac{13}{2} - \frac{7}{2} = 3 \text{ or } -10.$$

64.

$$2x^2 + 6x = 36;$$

$$x^2 + 3x = 18;$$

$$x^2 + 3x + \frac{9}{4} = 18 + \frac{9}{4} = \frac{81}{4};$$

$$x + \frac{3}{2} = \pm \frac{9}{2};$$

$$x = \pm \frac{9}{2} - \frac{3}{2} = 3 \text{ or } -6.$$

65.

$$5x^2 + 2x = 304;$$

$$x^2 + \frac{2x}{5} + \frac{1}{25} = \frac{304}{5} + \frac{1}{25} = \frac{1521}{25};$$

$$x + \frac{1}{5} = \pm \frac{39}{5};$$

$$x = \pm \frac{39}{5} - \frac{1}{5} = 7\frac{1}{5} \text{ or } -8.$$

66.

$$x^2 + 4x = 5;$$

$$x^2 + 4x + 4 = 9;$$

$$x + 2 = \pm 3;$$

$$x = \pm 3 - 2 = 1 \text{ or } -5.$$

67.

$$2x^2 - 2x = 40;$$

$$x^2 - x + \frac{1}{4} = 20 + \frac{1}{4} = \frac{81}{4};$$

$$x - \frac{1}{2} = \pm \frac{9}{2};$$

$$x = \pm \frac{9}{2} + \frac{1}{2} = 5 \text{ or } -4.$$

68.

$$2x^2 - 3x = 2;$$

$$x^2 - \frac{3x}{2} + \frac{9}{16} = 1 + \frac{9}{16} = \frac{25}{16};$$

$$x - \frac{3}{4} = \pm \frac{5}{4};$$

$$x = \pm \frac{5}{4} + \frac{3}{4} = 2 \text{ or } -\frac{1}{2}.$$

59.

$$x^2 + \frac{7x}{12} = \frac{31}{6};$$

$$x^2 + \frac{7x}{12} + \frac{49}{576} = \frac{31}{6} + \frac{49}{576} = \frac{2976 + 49}{576} = \frac{3025}{576};$$

$$x + \frac{7}{24} = \pm \frac{55}{24};$$

$$x = \pm \frac{55}{24} - \frac{7}{24} = 2 \text{ or } -2\frac{1}{2}.$$

60.

$$x^2 - \frac{2x}{3} = \frac{8}{3};$$

$$x^2 - \frac{2x}{3} + \frac{1}{9} = \frac{8}{3} + \frac{1}{9} = \frac{25}{9};$$

$$x - \frac{1}{3} = \pm \frac{5}{3};$$

$$x = \pm \frac{5}{3} + \frac{1}{3} = 2 \text{ or } -1\frac{1}{3}.$$

61.

$$x^2 - 5x = -6;$$

$$x^2 - 5x + \frac{25}{4} = -6 + \frac{25}{4} = \frac{1}{4};$$

$$x - \frac{5}{2} = \pm \frac{1}{2};$$

$$x = \pm \frac{1}{2} + \frac{5}{2} = 3 \text{ or } 2.$$

62.

$$x^2 - 3x = 4;$$

$$x^2 - 3x + \frac{9}{4} = 4 + \frac{9}{4} = \frac{25}{4};$$

$$x - \frac{3}{2} = \pm \frac{5}{2};$$

$$x = \pm \frac{5}{2} + \frac{3}{2} = 4 \text{ or } -1.$$

63.

$$x^2 + 7x = 80;$$

$$x^2 + 7x + \frac{49}{4} = 80 + \frac{49}{4} = \frac{169}{4};$$

$$x + \frac{7}{2} = \pm \frac{13}{2};$$

$$x = \pm \frac{13}{2} - \frac{7}{2} = 3 \text{ or } -10.$$

64.

$$2x^2 + 6x = 36;$$

$$x^2 + 3x = 18;$$

$$x^2 + 3x + \frac{9}{4} = 18 + \frac{9}{4} = \frac{81}{4};$$

$$x + \frac{3}{2} = \pm \frac{9}{2};$$

$$x = \pm \frac{9}{2} - \frac{3}{2} = 3 \text{ or } -6.$$

65.

$$5x^2 + 2x = 304;$$

$$x^2 + \frac{2x}{5} + \frac{1}{25} = \frac{304}{5} + \frac{1}{25} = \frac{1521}{25};$$

$$x + \frac{1}{5} = \pm \frac{39}{5};$$

$$x = \pm \frac{39}{5} - \frac{1}{5} = 7\frac{4}{5} \text{ or } -8.$$

66.

$$x^2 + 4x = 5;$$

$$x^2 + 4x + 4 = 9;$$

$$x + 2 = \pm 3;$$

$$x = \pm 3 - 2 = 1 \text{ or } -5.$$

67.

$$2x^2 - 2x = 40;$$

$$x^2 - x + \frac{1}{4} = 20 + \frac{1}{4} = \frac{81}{4};$$

$$x - \frac{1}{2} = \pm \frac{9}{2};$$

$$x = \pm \frac{9}{2} + \frac{1}{2} = 5 \text{ or } -4.$$

68.

$$2x^2 - 3x = 2;$$

$$x^2 - \frac{3x}{2} + \frac{9}{16} = 1 + \frac{9}{16} = \frac{25}{16};$$

$$x - \frac{3}{4} = \pm \frac{5}{4};$$

$$x = \pm \frac{5}{4} + \frac{3}{4} = 2 \text{ or } -\frac{1}{2}.$$

69.

$$x^2 - \frac{x}{2} = \frac{3}{2};$$

$$x^2 - \frac{x}{2} + \frac{1}{16} = \frac{8}{2} + \frac{1}{16} = \frac{25}{16};$$

$$x - \frac{1}{4} = \pm \frac{5}{4};$$

$$x = \pm \frac{5}{4} + \frac{1}{4} = 1\frac{1}{2} \text{ or } -1.$$

70.

$$x^2 + 3x = -2;$$

$$x^2 + 3x + \frac{9}{4} = -2 + \frac{9}{4} = \frac{1}{4};$$

$$x + \frac{3}{2} = \pm \frac{1}{2};$$

$$x = \pm \frac{1}{2} - \frac{3}{2} = -1 \text{ or } -2.$$

71.

$$x^2 - \frac{11x}{2} = \frac{21}{2};$$

$$x^2 - \frac{11x}{2} + \frac{121}{16} = \frac{21}{2} + \frac{121}{16} = \frac{289}{16};$$

$$x - \frac{11}{4} = \pm \frac{17}{4};$$

$$x = \pm \frac{17}{4} + \frac{11}{4} = 7 \text{ or } -1\frac{1}{2}.$$

72.

$$x^2 + x = 6;$$

$$x^2 + x + \frac{1}{4} = \frac{25}{4};$$

$$x + \frac{1}{2} = \pm \frac{5}{2};$$

$$x = \pm \frac{5}{2} - \frac{1}{2} = 2 \text{ or } -3.$$

73.

$$x^2 + \frac{8x}{3} + \frac{16}{9} = \frac{28}{3} + \frac{16}{9} = \frac{100}{9};$$

$$x + \frac{4}{3} = \pm \frac{10}{3};$$

$$x = \pm \frac{10}{3} - \frac{4}{3} = 2 \text{ or } -4\frac{2}{3}.$$

$$x^2 - \frac{3x}{2} + \frac{9}{16} = 27 + \frac{9}{16} = \frac{441}{16};$$

$$x - \frac{3}{4} = \pm \frac{21}{4};$$

$$x = \pm \frac{21}{4} + \frac{3}{4} = 6 \text{ or } -4\frac{1}{2}.$$

$$8x^2 - 8x - 6x^2 + 9 = x;$$

$$2x^2 - 9x = -9;$$

$$x^2 - \frac{9x}{2} + \frac{81}{16} = -\frac{9}{2} + \frac{81}{16} = \frac{9}{16};$$

$$x - \frac{9}{4} = \pm \frac{3}{4};$$

$$x = \pm \frac{3}{4} + \frac{9}{4} = 3 \text{ or } 1\frac{1}{2}.$$

$$7x - 14 - 5x^2 + 30x = 35x - 245;$$

$$-5x^2 + 2x = -231;$$

$$x^2 - \frac{2x}{5} = \frac{231}{5};$$

$$x^2 - \frac{2x}{5} + \frac{1}{25} = \frac{231}{5} + \frac{1}{25} = \frac{1156}{25};$$

$$x - \frac{1}{5} = \pm \frac{34}{5};$$

$$x = \pm \frac{34}{5} + \frac{1}{5} = 7 \text{ or } -6\frac{4}{5}.$$

$$4x^2 + 8 - 15x^2 + 25x = 12x - 16;$$

$$-11x^2 + 13x = -24;$$

$$x^2 - \frac{13x}{11} = \frac{24}{11};$$

$$x^2 - \frac{13x}{11} + \frac{169}{484} = \frac{24}{11} + \frac{169}{484} = \frac{1225}{484};$$

$$x - \frac{13}{22} = \pm \frac{35}{22};$$

$$x = \pm \frac{35}{22} + \frac{13}{22} = 2\frac{1}{11} \text{ or } -1.$$

78.

$$40x^2 - 30 - 12x^2 + 24x = 60x - 5;$$

$$28x^2 - 36x = 25;$$

$$x^2 - \frac{9x}{7} = \frac{25}{28};$$

$$x^2 - \frac{9x}{7} + \frac{81}{196} = \frac{25}{28} + \frac{81}{196} = \frac{175+81}{196} = \frac{256}{196};$$

$$x - \frac{9}{14} = \pm \frac{16}{14};$$

$$x = \pm \frac{16}{14} + \frac{9}{14} = \frac{25}{14} \text{ or } -\frac{7}{14} = 1\frac{1}{2} \text{ or } -\frac{1}{2}.$$

79.

$$35x^3 - 70x + 84x - 280x^3 = 16x - 60x^3 - 7;$$

$$-185x^3 - 2x = -7;$$

$$x^3 + \frac{2x}{185} + \frac{1}{185^3} = \frac{7}{185} + \frac{1}{185^3} = \frac{1296}{185^3};$$

$$x + \frac{1}{185} = \pm \frac{36}{185};$$

$$x = \pm \frac{36}{185} - \frac{1}{185} = \frac{7}{37} \text{ or } -\frac{1}{5}.$$

80.

$$108x^2 - 12 - 9x - 81x^2 = 36x^2 - 36x - 64;$$

$$-9x^2 + 27x = -52;$$

$$x^2 - 3x + \frac{9}{4} = \frac{52}{9} + \frac{9}{4} = \frac{289}{36};$$

$$x - \frac{3}{2} = \pm \frac{17}{6};$$

$$x = \pm \frac{17}{6} + \frac{3}{2} = 4\frac{1}{2} \text{ or } -1\frac{1}{2}.$$

EXERCISE XVIII.

1.

$$0 + 6 - 0 + 4 + 27 = 37.$$

2.

$$\begin{array}{r} 7x - 5y + 4z \\ x + 2y - 11z \\ 3x - y + 5z \\ 5x - 3y - z \\ \hline 16x - 7y - 3z \end{array}$$

3.

$$\begin{array}{r} 5a - 3b + 6c \\ 2a + 5b - 4c \\ \hline 3a - 8b + 10c \end{array}$$

$$\begin{array}{r}
 6a^2 + 5a^2b - 4ab^2 + 3b^3 \\
 a^2 - 2ab + b^2 \\
 \hline
 6a^5 + 5a^4b - 4a^3b^2 + 3a^2b^3 \\
 - 12a^4b - 10a^3b^2 + 8a^2b^3 - 6ab^4 \\
 \hline
 6a^5 - 7a^4b - 8a^3b^2 + 16a^2b^3 - 10ab^4 + 3b^5
 \end{array}$$

$$\begin{array}{r}
 x-1 \quad x^4 - 3x^3 + 2x^2 - x + 1 \quad (x^3 - 2x^2 - 1 \\
 \hline
 x^4 - x^3 \\
 \hline
 -2x^3 + 2x^2 \\
 \hline
 -2x^3 + 2x^2 \\
 \hline
 -x + 1 \\
 \hline
 -x + 1
 \end{array}$$

$$\begin{array}{r}
 x^3 - 1 \quad x^{10} - x \quad (x^7 + x^4 + x \\
 \hline
 x^{10} - x^7 \\
 \hline
 x^7 - x \\
 \hline
 x^7 - x^4 \\
 \hline
 x^4 - x \\
 \hline
 x^4 - x
 \end{array}$$

$$\begin{array}{r}
 x^6 - 4x^5 + 10x^4 - 12x^3 + 9x^2 \quad (x^3 - 2x^2 + 3x \\
 x^6 \\
 \hline
 2x^3 - 2x^2 \quad \begin{array}{l} -4x^5 + 10x^4 - 12x^3 + 9x^2 \\ -4x^5 + 4x^4 \end{array} \\
 \hline
 2x^3 - 4x^2 + 3x \quad \begin{array}{l} 6x^4 - 12x^3 + 9x^2 \\ 6x^4 - 12x^3 + 9x^2 \end{array}
 \end{array}$$

$$\begin{aligned}
 2a^3 + 3a^2 - [7a^2 - 10a + 2a^3 - 4a^2 + 12a] \\
 2a^3 + 3a^2 - 7a^2 + 10a - 2a^3 + 4a^2 - 12a = -2a.
 \end{aligned}$$

$$\begin{aligned}
 6x - 3 - 16x + 4 &= 12x - 54; \\
 6x - 16x - 12x &= -54 + 3 - 4; \\
 -22x &= -55; \\
 x &= \frac{55}{22} = 2\frac{1}{2}.
 \end{aligned}$$

Let x be the number,

n

$$2x + 32 = 10x.$$

$$2x - 10x - 32; \therefore -8x = -32, \text{ or } x = 4.$$

$$0 + 24 + 8 + 96 = 128.$$

$$\begin{array}{r}
 12. \quad 12a^2 + 5a^2 + a - 10 \\
 \quad \quad a^3 + 4a - 12 \\
 \quad \quad \quad 6a^2 + 7a \\
 \quad \quad \quad \quad 2a^2 + a + 9 \\
 \hline
 \quad \quad 13a^3 + 13a^2 + 13a - 18 \\
 = 13(a^3 + a^2 + a - 1)
 \end{array}$$

$$\begin{array}{r}
 13. \quad 4a^3 - 3a^2b + 2ab^2 + 9 \\
 \quad \quad a^3 - a^2b - 3ab^2 - b^3 + 9 \\
 \hline
 \quad \quad 3a^3 - 2a^2b + 5ab^2 + b^3 + 9
 \end{array}$$

$$\begin{array}{r}
 14. \quad x^3 + 3x^2 - 3x - 2 \\
 \quad \quad x^3 - 2x + 4 \\
 \hline
 \quad \quad x^6 + 3x^5 - 8x^4 - 2x^3 \\
 \quad \quad \quad - 2x^4 - 6x^3 + 6x^2 + 4x \\
 \quad \quad \quad \quad + 4x^3 + 12x^2 - 12x - 8 \\
 \hline
 \quad \quad x^6 + 3x^5 - 5x^4 - 4x^3 + 18x^2 - 8x - 8
 \end{array}$$

$$\begin{array}{r}
 15. \quad 3a + 4b \quad 6a^2 + 5ab - 4b^2 \quad (2a - b) \\
 \quad \quad \quad 6a^2 + 8ab \\
 \hline
 \quad \quad \quad - 8ab - 4b^2 \\
 \hline
 \quad \quad \quad - 8ab - 4b^2
 \end{array}$$

$$\begin{array}{r}
 16. \quad x^2 + xy + y^2 \quad x^6 - y^6 \quad (x^4 - x^2y + xy^2 - y^4) \\
 \quad \quad \quad x^6 + x^5y + x^4y^2 \\
 \hline
 \quad \quad \quad - x^5y - x^4y^2 - y^6 \\
 \quad \quad \quad - x^5y - x^4y^2 - x^3y^3 \\
 \hline
 \quad \quad \quad \quad x^3y^3 - y^6 \\
 \quad \quad \quad \quad x^3y^3 + x^2y^4 + xy^5 \\
 \hline
 \quad \quad \quad \quad \quad - x^2y^4 - xy^5 - y^6 \\
 \quad \quad \quad \quad \quad - x^2y^4 - xy^5 - y^6
 \end{array}$$

$$\begin{array}{r}
 17. \quad 4 - 4x - 11x^2 + 14x^3 + 5x^4 - 12x^5 + 4x^6 \quad (2 - x - 3x^2 + 2x^3) \\
 \quad \quad \quad 4 \\
 \hline
 \quad \quad 4 - x \quad \left| \begin{array}{r} -4x - 11x^2 + 14x^3 + 5x^4 - 12x^5 + 4x^6 \\ -4x + x^2 \end{array} \right. \\
 \hline
 \quad \quad 4 - 2x - 8x^2 \quad \left| \begin{array}{r} -12x^3 + 14x^3 + 5x^4 - 12x^5 + 4x^6 \\ -12x^3 + 6x^3 + 9x^4 \end{array} \right. \\
 \hline
 \quad \quad 4 - 2x - 6x^2 + 2x^3 \quad \left| \begin{array}{r} 8x^3 - 4x^4 - 12x^5 + 4x^6 \\ 8x^3 - 4x^4 - 12x^5 + 4x^6 \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 18. \quad (i) \quad 15x - 48x + 24 = 24x - 32 - x, \\
 \quad \quad \quad 15x - 48x - 24x + x = -32 - 24, \\
 \quad \quad \quad \quad -56x = -56, \text{ or } x = 1.
 \end{array}$$

$$\begin{array}{l}
 (ii) \quad 7x + 20 - 4x = 11x - 12, \\
 \quad \quad 7x - 4x - 11x = -12 - 20, \\
 \quad \quad \quad -8x = -32, \text{ or } x = 4.
 \end{array}$$

$$19. \quad 5x + 3y = 120,$$

$$10x - 9y = 90;$$

from (1),

$$10x + 6y = 240;$$

∴ by subtraction

$$-15y = -150, \text{ or } y = 10;$$

substitute in (1), then

$$5x + 30 = 120, \text{ or } x = 18.$$

20.

$$\begin{array}{r} 60^{\circ} 90' 24'' 16 (7804 \\ 49 \\ 148 \overline{) 1190} \\ \underline{1184} \\ 15604 \overline{) 62416} \\ \underline{62416} \end{array}$$

21.

$$\begin{aligned} \text{(i)} \quad & 48 - 18 + 48 - 16 = 62; \\ \text{(ii)} \quad & \frac{8+24}{12} + \frac{64-86}{6+8} - \frac{4+6+8}{3}; \\ & = 2\frac{2}{3} + 2 - 6 = -1\frac{1}{3}. \end{aligned}$$

22.

$$\begin{array}{r} 7a^3 - 2a^2b + 3ab^2 + 7b^3 \\ 11a^3b - 2ab^2 + 2b^3 \\ - 10a^3b \quad - 10b^3 \\ 11a^3 - 2a^2b \quad - b^3 \\ \hline 18a^3 - 3a^2b + ab^2 - 2b^3 \end{array}$$

23.

$$\begin{array}{r} 9x^3 - 2x^2y + 4y^3 \\ 7x^3 - 3x^2y + 5xy^2 - 6y^3 \\ \hline 2x^3 + x^2y - 5xy^2 + 10y^3 \end{array}$$

24.

$$\begin{array}{r} 3a^3 - 2a^2x - 4x^3 \\ 2a^3 - 7a^2x + x^3 \\ \hline 6a^3 - 4a^2x \\ - 21a^2x + 14a^2x^2 - 8a^2x^3 + 28ax^4 \\ \hline 3a^2x^2 - 2a^2x^3 - 4x^5 \\ \hline 6a^3 - 25a^2x + 17a^2x^2 - 10a^2x^3 + 28ax^4 - 4x^5 \end{array}$$

25.

$$\begin{array}{r} x^2 - 1 \overline{) 2x^3 + 3x^2 - 2x - 3} \quad (2x + 3 \\ \underline{2x^3} \quad \quad - 2x \\ \hline 3x^2 \quad \quad - 3 \\ 3x^2 \quad \quad - 3 \\ \hline \end{array}$$

26.

$$\begin{array}{r} a^3 + 2a + 1 \overline{) a^6 + 2a^5 + a^4} \quad (a^4 - 2a^3 + 3a^2 - 2a + 1 \\ \underline{a^6 + 2a^5 + a^4} \\ - 2a^5 - a^4 + 2a^3 + 1 \\ \underline{- 2a^5 - 4a^4 - 2a^3} \\ 3a^4 + 4a^3 + 1 \\ \underline{3a^4 + 6a^3 + 3a^2} \\ - 2a^3 - 3a^2 + 1 \\ \underline{- 2a^3 - 4a^2 - 2a} \\ a^2 + 2a + 1 \\ \underline{a^2 + 2a + 1} \end{array}$$

27.

$$\begin{array}{r} 100x^6 - 140x^5 + 129x^4 - 116x^3 + 58x^2 - 24x + 9 \overline{) 100x^6} \\ 20x^3 - 7x^2 \overline{) 100x^6 - 140x^5 + 129x^4} \\ \underline{100x^6 - 140x^5 + 49x^4} \\ 20x^3 - 14x^2 + 4x \overline{) 80x^4 - 116x^3 + 58x^2} \\ \underline{80x^4 - 56x^3 + 16x^2} \\ 20x^3 - 14x^2 + 8x - 3 \overline{) 60x^3 + 42x^2 - 24x + 9} \\ \underline{60x^3 + 42x^2 - 24x + 9} \end{array}$$

28. (i) $35x + 7x - 49 - 10x + 15 = 850;$
 $35x + 7x - 10x = 850 + 49 - 15;$
 $32x = 384; \therefore x = 12.$

(ii) $2x - y + 28 = 36, \text{ or } 2x - y = 8;$
 $2y + x + 48 = 57, \text{ or } x + 2y = 9.$

Multiply (1) by 2, and add to (2), then $5x = 25, \text{ or } x = 5;$
 substitute in (1), then $y = 2.$

29. Let x be the number;
 then $\frac{x}{5} - \frac{x}{8} = 7\frac{1}{2};$
 $8x - 5x = 300;$
 $3x = 300; \therefore x = 100.$

30. Multiply (1) by 2, then $6x - 2y + 2z = 34,$
 from (2) $5x + 3y - 2z = 10;$
 \therefore by addition $11x + y = 44 \dots\dots\dots (A).$

Multiply (1) by 5, then $15x - 5y + 5z = 85,$
 from (3) $7x + 4y - 5z = 3;$
 \therefore by addition $22x - y = 88 \dots\dots\dots (B).$

Add together (A) and (B), then $33x = 132, \text{ or } x = 4;$
 substitute in (A), then $44 + y = 44, \text{ or } y = 0;$
 substitute in (1), then $12 - 0 + z = 17, \text{ or } z = 5.$

31. $54 + 0 - 28 - 0 - 25 = 1.$

32.
$$\begin{array}{r} 3x^2 \qquad + y^2 - 2yz - z^2 \\ 2xy - 3y^2 + 3yz \\ -4x^2 - 2xy + y^2 + 2z^2 \\ \hline -x^2 \qquad - y^2 + yz + z^2 \end{array}$$

33.
$$\begin{array}{r} 2a^3 \qquad - 3ab^2 + 2b^3 \\ a^3 - 2a^2b + ab^2 + 4b^3 \\ \hline a^3 + 2a^2b - 4ab^2 - 2b^3 \end{array}$$

34.
$$\begin{array}{r} 3a^2 - 4ab + 2b^2 \\ 2a^2 - 4ab - 3b^2 \\ \hline 6a^4 - 8a^3b + 4a^2b^2 \\ - 12a^2b^2 + 16a^2b^2 - 8ab^3 \\ - 9a^3b^2 + 12ab^3 - 6b^4 \\ \hline 6a^4 - 20a^3b + 11a^2b^2 + 4ab^3 - 6b^4 \end{array}$$

$$\begin{array}{r}
 35. \quad 2x-7) 6x^4-17x^3-22x^2+26x+7(3x^3+2x^2-4x-1 \\
 \underline{6x^4-21x^3} \\
 4x^3-22x^2 \\
 \underline{4x^3-14x^2} \\
 -8x^2+26x \\
 \underline{-8x^2+28x} \\
 -2x+7 \\
 \underline{-2x+7}
 \end{array}$$

$$\begin{array}{r}
 36. \quad (i) \quad \begin{array}{r} a^2+2ab+b^2-c^2 \\ a^2-2ab+b^2-c^2 \\ \hline a^4+2a^3b+a^2b^2-a^2c^2 \\ -2a^3b-4a^2b^2 \qquad -2ab^3+2abc^2 \\ \qquad a^2b^2 \qquad +2ab^3+b^4-b^2c^2 \\ \qquad \qquad -a^2c^2 \qquad -2abc^2-b^2c^2+c^4 \\ \hline a^4 \qquad -2a^2b^2-2a^2c^2 \qquad + \qquad b^4-2b^2c^2+c^4 \end{array}
 \end{array}$$

(ii) The product $= \{(a+b)^3 - c^3\} \{(a-b)^3 - c^3\}$
 $= (a+b+c)(a+b-c)(a-b+c)(a-b-c):$
the divisor $= a^2 - (b-c)^2 = (a+b-c)(a-b+c).$

Dividing one by the other, we have

$$\begin{aligned}
 & (a+b+c)(a-b-c) \\
 & = a^2 - b^2 - 2bc - c^2.
 \end{aligned}$$

$$\begin{array}{r}
 37. \quad \begin{array}{r} 16-8a+a^2+40a^3-10a^4+25a^5(4-a+5a^2) \\ 16 \\ 8-a \overline{) 8a+a^2} \\ \underline{-8a+a^2} \\ 8-2a+5a^3 \end{array} \quad \begin{array}{r} 40a^3-10a^4+25a^5 \\ \underline{40a^3-10a^4+25a^5} \end{array}
 \end{array}$$

$$\begin{array}{r}
 38. \quad (i) \quad 14x+4-240-15x=12x+52-85x; \\
 14x-15x-12x+85x=52-4+240; \\
 72x=288; \therefore x=4.
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad 238x-966y=-1442, \\
 238x-221y=-697;
 \end{array}$$

\therefore by subtraction,

substitute in (1), then

$$\begin{aligned}
 & -745y = -745; \therefore y = 1; \\
 & 17x - 69 = -103; \\
 & \therefore 17x = -34, \text{ or } x = -2.
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & 5x^2+2y^2-[3x^2-2\{x^2-3y^2+4y^2-4x^2\}+x^2] \\
 & = 5x^2+2y^2-[3x^2-2x^2+6y^2-8y^2+8x^2+x^2] \\
 & = 5x^2+2y^2-3x^2+2x^2-6y^2+8y^2-8x^2-x^2 \\
 & = 6x^2+4y^2-11x^2.
 \end{aligned}$$

$$\begin{array}{r}
 40. \quad 6x^3 + 11x + 8) 12x^3 + 18x^2 - 14x - 21 (2x \\
 \underline{12x^3 + 22x^2 + 6x} \\
 -4x^2 - 20x - 21 \\
 8 \\
 \underline{-12x^2 - 60x - 68} (-2 \\
 \underline{-12x^2 - 22x - 6} \\
 -19) - 38x - 57 \\
 2x + 3) 6x^2 + 11x + 3 (3x + 1 \\
 \underline{6x^2 + 9x} \\
 2x + 3 \\
 \underline{2x + 3}
 \end{array}$$

Dividing numerator and denominator by $2x + 3$, we obtain $\frac{3x+1}{6x^2-7}$.

$$\begin{array}{ll}
 41. & (i) \quad 36 + 96 + 86 = 168; \\
 & (ii) \quad 540 - (25 - 16) - 8(0 + 64) \\
 & \quad \quad = 540 - 9 - 192 = 339.
 \end{array}$$

$$\begin{array}{r}
 42. \quad \begin{array}{r} 3x^3 - 2xy^2 + y^3 \\ 2x^3 - 3x^2y - 2xy^2 + 3y^3 \\ \hline 2xy^2 - 4y^3 \\ 5x^3 - 2xy^3 \end{array}
 \end{array}$$

$$\begin{array}{l}
 43. \quad 12a - [4a + \{3a - 2b - 4b + 5a\}] \\
 = 12a - [4a + 3a - 2b - 4b + 5a] \\
 = 12a - 4a - 8a + 2b + 4b - 5a = 6b.
 \end{array}$$

$$\begin{array}{r}
 44. \quad \begin{array}{r} 2x^3 - 3xy + 5y^3 \\ 2x^3 + 3xy - 5y^3 \\ \hline 4x^4 - 6x^2y + 10x^2y^3 \\ + 6x^3y - 9x^3y^3 + 15xy^3 \\ - 10x^2y^3 + 15xy^3 - 25y^4 \\ \hline 4x^4 - 9x^2y^2 + 30xy^3 - 25y^4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 45. \quad 3x^3 - 7x + 2) 6x^4 - 23x^3 + 13x^2 + 22x - 8 (2x^2 - 3x - 4 \\
 \underline{6x^4 - 14x^3 + 4x^2} \\
 -9x^3 + 9x^2 + 22x \\
 \underline{-9x^3 + 21x^2 - 6x} \\
 -12x^2 + 28x - 8 \\
 \underline{-12x^2 + 28x - 8}
 \end{array}$$

$$\begin{array}{r}
 46. \quad a^3 - 2a^2b + 2ab^2 - b^3) a^6 - b^6 \\
 \underline{a^6 - 2a^5b + 2a^4b^2 - a^3b^3} \\
 2a^5b - 2a^4b^2 + a^3b^3 - b^6 \\
 \underline{2a^5b - 4a^4b^2 + 4a^3b^3 - 2a^2b^4} \\
 2a^4b^2 - 8a^3b^3 + 2a^2b^4 - b^6 \\
 \underline{2a^4b^2 - 4a^3b^3 + 4a^2b^4 - 2ab^5} \\
 a^3b^3 - 2a^2b^4 + 2ab^5 - b^6 \\
 \underline{a^3b^3 - 2a^2b^4 + 2ab^5 - b^6}
 \end{array}$$

$$\begin{array}{r}
 47. \quad \begin{array}{r} 2x^3 \quad \quad + 4a^2x - a^3 \\ + 2ax^2 \quad + 8a^3 \\ \hline x^3 - ax^2 - a^2x \\ 3x^3 + ax^2 + 8a^2x + 8a^3 \end{array} \quad \begin{array}{r} x^3 + 3ax^2 + a^3 \\ \hline x^3 - a^3 \\ 3ax^2 + 2a^3 \end{array} \\
 \hline
 \begin{array}{r} 3x^3 + ax^2 + 8a^2x + 8a^3 \\ 3x^3 + ax^2 + 8a^2x + 2a^3 \\ \hline 8a^3 \\ 8a^3 + 2a^3 \\ \hline 3x^3 - 2ax^2 + 8a^2x \end{array}
 \end{array}$$

$$\begin{aligned}
 48. \quad & 4x^3 - 3x^2 - \{2x^3 - 6x - 15 + 14x^2 - 10x + 18\} \\
 & = 4x^3 - 3x^2 - 2x^3 + 6x + 15 - 14x^2 + 10x - 18 \\
 & = 2x^3 - 17x^2 + 16x - 3.
 \end{aligned}$$

$$\begin{aligned}
 49. \quad (i) \quad & 63x + 77 - 14x - 2x + 4 = 504; \\
 & 63x - 14x - 2x = 504 - 77 - 4; \\
 & 47x = 423; \therefore x = 9.
 \end{aligned}$$

(ii) Equation (1) becomes

$$\begin{aligned}
 40x - 5y - 15 &= 140 + 12y - 8x, \\
 48x - 17y &= 155;
 \end{aligned}$$

$$\begin{aligned}
 \text{Equation (2) becomes } 15y + x - 2 &= 78, \\
 x + 15y &= 80.
 \end{aligned}$$

$$\begin{aligned}
 \text{From (1)} \quad & 48x - 17y = 155, \\
 \text{from (2)} \quad & 48x + 720y = 3840; \\
 \therefore -737y &= -3685, \text{ or } y = 5; \\
 \text{substitute in (1), then} \quad & x + 75 = 80, \text{ or } x = 5.
 \end{aligned}$$

$$\begin{aligned}
 50. \quad & \text{Let } x = C's \text{ share in } \pounds's, \\
 & \text{then } x + 21 = B's \text{ share in } \dots, \\
 & \text{and } x + 36 = A's \text{ share in } \dots; \\
 & \text{hence } x + x + 21 + x + 36 = 129, \\
 & 3x = 129 - 57 = 72; \therefore x = 24; \\
 & \text{hence } A \text{ gave } \pounds 60.
 \end{aligned}$$

$$51. \quad 40 - 112 - 50 + 128 + 0 = 6.$$

$$\begin{array}{r}
 52. \quad \begin{array}{r} 11a^2 - 3ab - 3ac + 2b^2 \\ 7a^2 - 3ab + 3ac - b^2 \\ \hline 4a^2 \quad \quad - 6ac + 3b^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 53. \quad \begin{array}{r} 2a^3 - 2a^2b - 4ab^2 + b^3 \\ 2a^3 - ab + 3b^3 \\ \hline 4a^3 - 4a^2b - 8a^2b^2 + 2a^2b^3 \\ - 2a^4b + 2a^3b^2 + 4a^2b^3 - ab^4 \\ \hline 6a^3b^2 - 6a^2b^3 - 12ab^4 + 3b^5 \\ \hline 4a^5 - 6a^4b \quad \quad - 18ab^4 + 3b^5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 54. \quad 1 - 2x + x^3 \quad 1 - 6x^5 + 5x^6 (1 + 2x + 3x^2 + 4x^3 + 5x^4) \\
 \hline
 1 - 2x + x^3 \\
 2x - x^2 - 6x^5 + 5x^6 \\
 \hline
 2x - 4x^3 + 2x^3 \\
 3x^3 - 2x^3 - 6x^5 + 5x^6 \\
 \hline
 3x^3 - 6x^3 + 3x^4 \\
 4x^3 - 3x^4 - 6x^5 + 5x^6 \\
 \hline
 4x^3 - 8x^4 + 4x^5 \\
 5x^4 - 10x^5 + 5x^6 \\
 \hline
 5x^4 - 10x^5 + 5x^6
 \end{array}$$

$$\begin{array}{l}
 55. \quad 2[x^3 - 3x + 3 + 5] - 3[x^2 - \{x - 2x^2 + 2\} + 2x] \\
 = 2x^3 - 6x + 6 + 10 - 3[x^2 - x + 2x^2 - 2 + 2x] \\
 = 2x^3 - 6x + 6 + 10 - 3x^2 + 3x - 6x^2 + 6 - 6x \\
 = -7x^2 - 9x + 22.
 \end{array}$$

$$\begin{array}{l}
 56. \quad 4x - 8 - 2 + x = 30; \\
 \quad \quad \quad 5x = 40; \therefore x = 8.
 \end{array}$$

$$\begin{array}{l}
 57. \text{ From (1),} \quad 3x - 2y = 11, \\
 \text{from (2),} \quad 4x + 2y = 24; \\
 \quad \quad \quad \therefore 7x = 35, \text{ or } x = 5; \\
 \text{substitute in (2), then} \quad 10 + y = 12, \text{ or } y = 2.
 \end{array}$$

$$\begin{array}{r}
 58. \quad (i) \quad \begin{array}{r} 9a^4 - 12a^3b + 22a^2b^2 - 12ab^3 + 9b^4 (3a^2 - 2ab + 3b^2) \\ 9a^4 \\ \hline 6a^2 - 2ab \quad \begin{array}{l} -12a^3b + 22a^2b^2 \\ -12a^3b + 4a^2b^2 \end{array} \\ \hline 6a^2 - 4ab + 3b^2 \quad \begin{array}{l} 18a^2b^2 - 12ab^3 + 9b^4 \\ 18a^2b^2 - 12ab^3 + 9b^4 \end{array} \end{array}
 \end{array}$$

$$(ii) \quad \begin{array}{r} 31'67'43'84 (562'8 \\ 25 \end{array}$$

$$\begin{array}{r}
 106 \overline{) 667} \\
 \underline{636} \\
 1122 \overline{) 3143} \\
 \underline{2244} \\
 11248 \overline{) 89984} \\
 \underline{89984}
 \end{array}$$

$$\begin{array}{r}
 59. \quad \begin{array}{r} 2x^2 - x - 36 \overline{) 6x^2 - 29x + 9} (3 \\ \underline{6x^2 - 3x - 108} \\ -13 \overline{) -26x + 117} \\ \underline{-13 \overline{) -26x + 117}} \\ 2x - 9 \overline{) 2x^2 - x - 36} (x + 4 \\ \underline{2x^2 - 9x} \\ 8x - 36 \\ \underline{8x - 36} \end{array} \\
 \text{a. c. m. } 2x - 9.
 \end{array}$$

60. Let
and
then

$$\begin{aligned}x &= \text{the son's age in years,} \\ 2x &= \text{the father's age} \\ 2x - 20 &= 4(x - 20) = 4x - 80, \\ \therefore -2x &= -60, \text{ or } x = 30.\end{aligned}$$

61.

$$\begin{array}{r} 3a^2 + 4ab + 5c^2 - 6bc - 8b^2 + 2ac \\ - 5a^2 - 3ab + c^2 + 3b^2 - 4ac \\ \hline 2a^2 - ab - 6c^2 + 6bc + 5b^2 + 2ac \end{array}$$

62.

$$\begin{aligned} \text{(i)} \quad & \frac{x^2 + 2xy - 3y^2}{x^2 - 5xy + 4y^2} \\ & \frac{x^4 + 2x^2y - 3x^2y^2 - 5x^2y^2 - 10x^2y^3 + 15xy^3}{+ 4x^2y^2 + 8xy^3 - 12y^4} \\ & \frac{x^4 - 3x^2y - 9x^2y^2 + 23xy^3 - 12y^4}{x^4 - 3x^2y - 9x^2y^2 + 23xy^3 - 12y^4} \\ \text{(ii)} \quad & \frac{a^4 + a^3 + a^2 + a + 1}{a - 1} \\ & \frac{a^5 + a^4 + a^3 + a^2 + a}{- a^4 - a^3 - a^2 - a - 1} \\ & \frac{a^5}{-1} \end{aligned}$$

$$\begin{array}{r} 63. \quad 3a^2 - 2ab + b^2 \quad 12a^4 - 26a^3b - 8a^2b^2 + 10ab^3 - 8b^4 \quad (4a^2 - 6ab - 8b^2) \\ \hline 12a^4 - 8a^3b + 4a^2b^2 \\ - 18a^3b - 12a^2b^2 + 10ab^3 \\ - 18a^3b + 12a^2b^2 - 6ab^3 \\ \hline - 24a^2b^3 + 16ab^3 - 8b^4 \\ - 24a^2b^3 + 16ab^3 - 8b^4 \end{array}$$

$$\begin{aligned} 64. \quad & \{2a^2 - 3ab + b^2\} - \{a^2 - 4ab - b^2\} + \{2b^2 - a^2 + ab\} \\ & = 2a^2 - 3ab + b^2 - a^2 + 4ab + b^2 + 2b^2 - a^2 + ab \\ & = 2ab + 4b^2. \end{aligned}$$

65. (i)

$$\begin{aligned} 60x - 25x + 20 &= 210 - 6 + 12x, \\ 60x - 25x - 12x &= 210 - 20 - 6, \\ 23x &= 184; \therefore x = 8. \end{aligned}$$

(ii) Equation (1) becomes $7x - y + 2 = 35$,

$$7x - y = 33;$$

equation (2) becomes

$$\begin{aligned} 12y - x - 10 &= 9, \\ -x + 12y &= 19; \end{aligned}$$

but from (1)

$$84x - 12y = 396;$$

\therefore by addition

$$83x = 415, \text{ or } x = 5;$$

substitute in (1), then

$$35 - y = 33, \text{ or } y = 2.$$

$$\begin{array}{r}
 66. \quad 2x^3 + x - 28 \overline{) 6x^3 - 21x^2 - 4x + 14} \quad (3x - 12) \\
 \underline{6x^3 + 3x^2 - 84x} \\
 -24x^2 + 80x + 14 \\
 \underline{-24x^2 - 12x + 836} \\
 46 \overline{) 92x - 322} \\
 \underline{2x - 7} \quad 2x^2 + x - 28 \quad (x + 4) \\
 \underline{2x^2 - 7x} \\
 8x - 28 \\
 \underline{8x - 28} \\
 0
 \end{array}$$

Dividing numerator and denominator by $2x - 7$, we have $\frac{x+4}{3x^2-2}$.

$$\begin{array}{r}
 67. \quad 3x^3 - 8x^2 - x + 10 \overline{) 3x^3 - 5x^2 - 3x + 5} \quad (1) \\
 \underline{3x^3 - 8x^2 - x + 10} \\
 3x^3 - 2x - 5 \overline{) 3x^3 - 8x^2 - x + 10} \quad (x - 2) \\
 \underline{3x^3 - 2x^2 - 5x} \\
 -6x^2 + 4x + 10 \\
 \underline{-6x^2 + 4x + 10} \\
 0
 \end{array}$$

$3x^3 - 2x - 5$ is G. C. M., and dividing both by it we obtain as L. C. M.
 $(3x^3 - 2x - 5)(x - 2)(x - 1)$.

$$\begin{array}{ll}
 68. \text{ From (1),} & 2x + 4y + 6z = 34, \\
 \text{from (2),} & 2x - 3y + z = 0, \\
 \therefore \text{ by subtraction,} & 7y + 5z = 34 \dots\dots\dots (A); \\
 \text{again, from (1),} & 3x + 6y + 9z = 51, \\
 \text{from (2),} & 3x + y - 5z = -15, \\
 \therefore \text{ by subtraction,} & 5y + 14z = 66 \dots\dots\dots (B). \\
 \text{Multiply (A) by 5,} & 35y + 25z = 170, \\
 \text{multiply (B) by 7,} & 35y + 98z = 462, \\
 \therefore \text{ by subtraction,} & -73z = -292, \text{ or } z = 4; \\
 \text{substitute in (A), then} & 7y + 20 = 34, \text{ or } y = 2; \\
 \text{substitute in (1), then} & x + 4 + 12 = 17, \text{ or } x = 1.
 \end{array}$$

$$\begin{array}{ll}
 69. \text{ Let } x \text{ be the greater part,} & \\
 \text{then } 84 - x \text{ is the less;} & \\
 \text{hence} & 2x = 5(84 - x), \\
 \therefore 2x = 420 - 5x; \text{ or } x = 60; & \\
 \text{hence 60 and 24 are the parts.} &
 \end{array}$$

$$\begin{array}{ll}
 70. \text{ Let } x \text{ and } y \text{ be the numbers;} & \\
 \text{then} & x + 2y = 24, \\
 \text{and} & y + 2x = 27; \\
 \text{multiply (1) by 2, then} & 2x + 4y = 48, \\
 \therefore \text{ by subtraction} & -3y = -21, \text{ or } y = 7; \\
 \text{substitute in (1), then} & x + 14 = 24, \text{ or } x = 10.
 \end{array}$$

71.

$$\begin{array}{r}
 x^3 - 3ax^2 + 3a^2x - a^3 \\
 4x^3 - 5ax^2 + 6a^2x - 15a^3 \\
 8x^3 + 4ax^2 + 2a^2x + 6a^3 \\
 - 17x^3 + 19ax^2 - 15a^2x + 8a^3 \\
 - 13ax^2 - 27a^2x + 18a^3 \\
 \hline
 - 9x^3 + 2ax^2 - 31a^2x + 16a^3
 \end{array}$$

72.

$$\begin{array}{r}
 -x^3 - 3xy + 3y^3 \\
 3x^3 + 4xy - 5y^3 \\
 \hline
 2x^3 + xy - 2y^3
 \end{array}
 \qquad
 \begin{array}{r}
 3x^3 + 2xy - y^3 \\
 2x^3 + xy - 2y^3 \\
 \hline
 x^3 + xy + y^3
 \end{array}$$

73. (i)

$$\begin{array}{r}
 a^2 + 2a - 1 \\
 a^2 - a + 1 \\
 \hline
 a^4 + 2a^3 - a^2 \\
 - a^3 - 2a^2 + a \\
 \hline
 a^2 + 2a - 1 \\
 \hline
 a^4 + a^3 - 2a^2 + 3a - 1
 \end{array}$$

(ii)

$$\begin{array}{r}
 9a^3 - 3ab + b^3 - 6a - 2b + 4 \\
 3a + b + 2 \\
 \hline
 27a^3 - 9a^2b + 3ab^2 - 18a^2 - 6ab + 12a \\
 9a^2b - 3ab^2 + b^3 - 6ab - 2b^2 + 4b \\
 \hline
 18a^3 - 6ab + 2b^3 - 12a - 4b + 8 \\
 \hline
 27a^3 + b^3 - 18ab + 8
 \end{array}$$

74.

$$\begin{array}{r}
 (i) \quad 2a - 4b \quad 6a^2 - 16ab + 8b^2 \quad (3a - 2b) \\
 \hline
 6a^3 - 12ab \\
 - 4ab + 8b^2 \\
 \hline
 - 4ab + 8b^2 \\
 \hline
 (ii) \quad 2x^2y^2 - 2xy + 1 \quad 4x^4y^4 + 1 \quad (2x^2y^2 + 2xy + 1) \\
 \hline
 4x^4y^4 - 4x^2y^2 + 2x^2y^2 \\
 \hline
 4x^2y^2 - 2x^2y^2 + 1 \\
 \hline
 4x^2y^2 - 4x^2y^2 + 2xy \\
 \hline
 2x^2y^2 - 2xy + 1 \\
 \hline
 2x^2y^2 - 2xy + 1
 \end{array}$$

75.

$$\begin{array}{r}
 x^6 - 4x^5y + 8x^4y^2 - 10x^3y^3 + 8x^2y^4 - 4xy^5 + y^6 \quad (x^3 - 2x^2y + 2xy^2 - y^3) \\
 \hline
 2x^3 - 2x^2y \quad \begin{array}{|l} -4x^2y + 8x^4y^2 \\ -4x^2y + 4x^4y^2 \end{array} \\
 \hline
 2x^3 - 4x^2y + 2xy^2 \quad \begin{array}{|l} 4x^4y^2 - 10x^3y^3 + 8x^2y^4 \\ 4x^4y^2 - 8x^3y^3 + 4x^2y^4 \end{array} \\
 \hline
 2x^3 - 4x^2y + 4xy^2 - y^3 \quad \begin{array}{|l} -2x^2y^3 + 4x^2y^4 - 4xy^5 + y^6 \\ -2x^2y^3 + 4x^2y^4 - 4xy^5 + y^6 \end{array}
 \end{array}$$

76.

$$6x - 3y = -9; \quad 3x + y = 13.$$

From (2)

$$6x + 2y = 26;$$

 \therefore by subtraction

$$-5y = -35, \text{ or } y = 7;$$

substitute in (2),

$$3x = 6, \text{ or } x = 2.$$

77. Multiply (3) by 4, $16x + 20y + 4z = 82$,
 and from (1), $2x + 8y + 4z = 1$,
 \therefore by subtraction $14x + 17y = 31$ (A).

Again, multiply (3) by 5, $20x + 25y + 5z = 40$,
 and from (2), $3x + 4y + 5z = 2$,
 \therefore by subtraction $17x + 21y = 38$ (B).

Multiply (A) by 21, $294x + 357y = 651$,
 and (B) by 17, $289x + 357y = 646$,
 \therefore by subtraction $5x = 5$, or $x = 1$;
 substitute in (A), $17y = 17$, or $y = 1$;
 substitute in (1), $2 + 3 + 4z = 1$, or $z = -1$.

78.

$$\begin{array}{r} .00'02'(.01414 \dots) \\ 1 \\ 24 \overline{) 100} \\ \underline{96} \\ 281 \\ 281 \overline{) 400} \\ \underline{281} \\ 2824 \overline{) 11900} \\ \underline{11296} \end{array}$$

79. Let x be A's share in £s;
 then $500 - x$ is B's

$$\therefore \frac{x}{5} = \frac{1}{3}(500 - x);$$

$$3x = 2500 - 5x; \text{ or } x = £312. 10s.,$$

and B's share is £187. 10s.

80. Let x feet be the breadth of the room,
 then $2x$ feet is its length;

hence $(x + 9)(2x - 10) = 2x^2$;
 $\therefore 8x = 90$, or $x = 11\frac{1}{2}$,

hence the length is $22\frac{1}{2}$ ft.

81.
$$\frac{300}{3} + \frac{75}{10} - 0 - \frac{150 - 81}{12}$$

$$= 100 + 7\frac{1}{2} - 5\frac{3}{4} = 101\frac{3}{4}.$$

82.
$$\begin{array}{r} a^3 - 6a^2b - 5ab^2 \\ 3a^3 - ab \\ \hline 3a^5 - 18a^4b - 15a^3b^2 \\ - a^4b + 6a^3b^2 + 5a^2b^3 \\ \hline 3a^5 - 19a^4b - 9a^3b^2 + 5a^2b^3 \end{array}$$

83.

$$\begin{array}{r}
 x^3 - 5y \\
 x^3 - 5y \\
 \hline
 x^4 - 5x^2y \\
 - 5x^2y + 25y^2 \\
 \hline
 x^4 - 10x^2y + 25y^2 \\
 x^3 - 5y \\
 \hline
 x^5 - 10x^3y + 25x^2y^2 \\
 - 5x^4y + 50x^2y^2 - 125y^3 \\
 \hline
 x^5 - 15x^4y + 75x^2y^2 - 125y^3
 \end{array}$$

84.

$$\begin{array}{r}
 4a^4 - 12a^3 + 13a^2 - 6a + 1 \quad (2a^2 - 3a + 1) \\
 4a^4 \\
 \hline
 4a^2 - 3a \quad \begin{array}{r} -12a^3 + 13a^2 \\ -12a^3 + 9a^2 \end{array} \\
 4a^2 - 6a + 1 \quad \begin{array}{r} 4a^2 - 6a + 1 \\ 4a^2 - 6a + 1 \end{array}
 \end{array}$$

85.

$$\begin{aligned}
 & a^3 - 12a^2 + 9ab - 3b^2 - 6\{b^3 - 2a^2 + 2ab - 3ab\} \\
 & = a^3 - 12a^2 + 9ab - 3b^2 - 6b^3 + 12a^2 - 12ab + 18ab \\
 & = a^3 + 15ab - 9b^3.
 \end{aligned}$$

86.

$$\begin{array}{r}
 3x^2 - 5x + 2 \quad 3x^3 - 4x^2 + 1 \quad (x \\
 3x^3 - 5x^2 + 2x \\
 \hline
 x^2 - 2x + 1 \quad 3x^2 - 5x + 2 \quad (3 \\
 3x^2 - 6x + 3 \\
 \hline
 x - 1 \quad x^2 - 2x + 1 \quad (x - 1 \\
 x^2 - x \\
 \hline
 -x + 1 \\
 -x + 1 \\
 \hline
 \end{array}$$

$x - 1$ is c. c. m.; divide both expressions by it, and we find as l. c. m.

$$(x - 1)(3x - 2)(3x^2 - x - 1).$$

87. (i)

$$\begin{aligned}
 14x + 28 &= 35x - 70 - 10x + 10; \\
 -11x &= -88, \text{ or } x = 8.
 \end{aligned}$$

(ii) From (1),

$$15x + 6y = 48,$$

from (2),

$$14x - 6y = 10;$$

 \therefore by addition

$$29x = 58, \text{ or } x = 2;$$

substitute in (1),

$$10 + 2y = 16, \text{ or } y = 3.$$

(iii) Subtract (1) from (3), then $2x = 40$, or $x = 20$;

subtract (2) from (1), then

$$2y = 20, \text{ or } y = 10;$$

add (2) and (3), then

$$2z = 10, \text{ or } z = 5.$$

D. A. K.

88.

$$\begin{array}{r}
 23 \cdot 4256 \text{ (4.84)} \\
 16 \\
 88 \overline{) 742} \qquad \qquad \qquad 4 \cdot 84 \text{ (2.2)} \\
 \underline{704} \qquad \qquad \qquad \underline{4} \\
 964 \overline{) 8856} \qquad \qquad \qquad 42 \overline{) 84} \\
 \underline{8856} \qquad \qquad \qquad \underline{84}
 \end{array}$$

89.

$$\begin{array}{r}
 3x^4 - 11x^2 + 10 \overline{) 6x^4 - 3x^3 - 22x^2 + 5x + 20} \\
 \underline{6x^4} \qquad \qquad \underline{- 22x^2} \qquad \underline{+ 20} \\
 -x - 3x^3 \qquad \qquad \qquad + 5x \\
 \qquad \qquad \qquad \underline{8x^3 - 5} \qquad \qquad \qquad 3x^4 - 11x^2 + 10 \overline{) (x^2 - 2)} \\
 \qquad \qquad \qquad \qquad \qquad \underline{3x^4 - 5x^2} \\
 \qquad \qquad \qquad \qquad \qquad \qquad \underline{- 6x^2 + 10} \\
 \qquad \qquad \qquad \qquad \qquad \qquad \underline{- 6x^2 + 10}
 \end{array}$$

c. c. m. is $3x^2 - 5$, and dividing numerator and denominator by it, we obtain

$$\frac{x^2 - 2}{2x^2 - x - 4}$$

90.

$$1 + 4 + 16 = 21; \quad (1 + 2 + 4)^2 = 7^2 = 49; \\
 \therefore \text{ difference is } 28.$$

91.

$$\begin{array}{r}
 3x^3 + 7x^2y - 3xy^2 + y^3 \\
 3x^2y \qquad \qquad \qquad - 3y^3 \\
 \underline{4x^3} \qquad \qquad \underline{- 8xy^2 + y^3} \\
 7x^3 + 11xy^2 + y^3 \\
 \underline{7x^3 + 17x^2y}
 \end{array}$$

92.

$$\begin{array}{r}
 9a^3b^3 + a^2b^2 - ab - 2 \\
 7a^3b^3 - 3a^2b^2 + 4ab - 7 \\
 \underline{2a^3b^3 + 4a^2b^2 - 5ab + 5}
 \end{array}$$

93.

$$\begin{array}{r}
 7a^2 - 3ab + 2b^2 \\
 2a^2 - 4ab - 3b^2 \\
 \underline{14a^4 - 6a^3b + 4a^2b^2} \\
 - 28a^3b + 12a^2b^2 - 8ab^3 \\
 - 21a^2b^3 + 9ab^3 - 6b^4 \\
 \underline{14a^4 - 34a^3b - 5a^2b^2 + ab^3 - 6b^4}
 \end{array}$$

94.

$$\begin{array}{r}
 a^2 + 2ab + b^2 \overline{) a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4} \\
 \underline{a^4 + 2a^3b + a^2b^2} \\
 2a^3b + 5a^2b^2 + 4ab^3 \\
 \underline{2a^3b + 4a^2b^2 + 2ab^3} \\
 a^2b^2 + 2ab^3 + b^4 \\
 \underline{a^2b^2 + 2ab^3 + b^4}
 \end{array}$$

$$95. \quad 2a - x^2) 64a^6 - x^{12} \quad (32a^5 + 16a^4x^2 + 8a^3x^4 + 4a^2x^6 + 2ax^8 + x^{10}$$

$$\begin{array}{r} 64a^6 - 32a^5x^2 \\ \hline 32a^5x^2 - x^{12} \\ \hline 32a^5x^2 - 16a^4x^4 \\ \hline 16a^4x^4 - x^{12} \\ \hline 16a^4x^4 - 8a^3x^6 \\ \hline 8a^3x^6 - x^{12} \\ \hline 8a^3x^6 - 4a^2x^8 \\ \hline 4a^2x^8 - x^{12} \\ \hline 4a^2x^8 - 2ax^{10} \\ \hline 2ax^{10} - x^{12} \\ \hline 2ax^{10} - x^{12} \end{array}$$

$$96. \quad 25 - [2^2 - \{1^2 - (-4)^2\}] \\ = 25 - [4 - \{1 - 16\}] = 25 - [4 + 15] = 6.$$

$$97. \quad a^6 + 4a^5 + 2a^4 + 9a^3 - 4a + 4(a^3 + 2a^2 - a + 2)$$

$$\begin{array}{r} 2a^3 + 2a^2 \overline{) a^6 + 4a^5 + 2a^4 + 9a^3 - 4a + 4} \\ \underline{4a^5 + 2a^4} \\ 2a^3 + 4a^2 - a \\ \underline{4a^2 + 4a^4} \\ -2a^4 + 9a^3 \\ \underline{-2a^4 - 4a^3 + a^2} \\ 2a^3 + 4a^2 - 2a + 2 \\ \underline{4a^3 + 8a^2 - 4a + 4} \\ 4a^3 + 8a^2 - 4a + 4 \end{array}$$

$$98. \quad (i) \quad 8x - 15 - x^2 + x^3 + x - 42 - 6x + 22 + 22 = 0; \\ 3x = 13; \therefore x = 4\frac{1}{3}.$$

$$(ii) \quad 60x - 20x + 40 = 15x + 345 - 120 - 12x; \\ 37x = 185; \therefore x = 5.$$

99. (i) From (1) $2x + 10 = 3y - 3$, or $2x - 3y = -13$;
from (2), $6x - y = 21$;
multiply (1) by 3, and subtract (2); $-8y = -60$; $\therefore y = 7\frac{1}{2}$;
substitute in (2); $6x = 21 + 7\frac{1}{2}$; $\therefore x = 4\frac{3}{4}$.

(ii) From (1) $3x + 6y - 9z = 3$,
and from (2) $3x - y + z = 2$,
 \therefore by subtraction $7y - 10z = 1$(A).

Again, from (1) $5x + 10y - 15z = 5$,
and from (3) $5x + 4y - 7z = 6$,
 \therefore by subtraction $6y - 8z = -1$(B).

Multiply (A) by 4, and (B) by (5); then $28y - 40z = 4$,
 $30y - 40z = -5$;

hence $-2y = 9$, or $y = -4\frac{1}{2}$;
substitute in (A), then $-31\frac{1}{2} - 10z = 1$, $\therefore z = -3\frac{1}{4}$;
substitute in (1), then $x - 9 + 9\frac{1}{4} = 1$; $\therefore x = \frac{1}{4}$.

100. Let $10x+y$ be the number;

then

$$y = x - 3,$$

and

$$10x + y = 6(x + y) + 7;$$

from (1)

$$4x - 4y = 12,$$

from (2)

$$4x - 5y = 7;$$

hence

$$y = 5, \text{ and } x = 8,$$

and the number is 85.

101.

$$\begin{array}{r} 9x^3 - 3xy + y^3 \\ 3x^2 - 2xy + y^2 \\ \hline 27x^4 - 9x^2y + 3x^2y^3 \\ - 18x^3y + 6x^2y^2 - 2xy^3 \\ 9x^2y^2 - 3xy^3 + y^4 \\ \hline 27x^4 - 27x^3y + 18x^2y^2 - 5xy^3 + y^4 \end{array}$$

102.

$$\begin{array}{r} x + 2a \quad 3x^3 + 4ax^2 - 6a^2x - 4a^3 \quad (3x^2 - 2ax - 2a^2) \\ \hline 3x^3 + 6ax^2 \\ \hline - 2ax^2 - 6a^2x \\ \hline - 2ax^2 - 4a^2x \\ \hline - 2a^2x - 4a^3 \\ \hline - 2a^2x - 4a^3 \end{array}$$

103.

$$\begin{aligned} a^3 - a^2 - 7a - \{a^3 - 6a - 2\} + \{a^3 - a - 7\} \\ = a^3 - a^2 - 7a - a^3 + 6a + 2 + a^3 - a - 7 \\ = a^3 - a^2 - 2a - 5. \end{aligned}$$

104. Divide the second expression by x^2 ; then

$$\begin{array}{r} 2x^2 + x - 6 \quad 8x^2 - 26x + 21 \quad (4 \\ \hline 8x^2 + 4x - 24 \\ \hline -15) - 30x + 45 \\ \hline 2x - 3 \quad 2x^2 + x - 6 \quad (x + 2 \\ \hline 2x^2 - 3x \\ \hline 4x - 6 \\ \hline 4x - 6 \end{array}$$

G. C. M. is $2x - 3$: dividing both expressions by it, we obtain as L. C. M.

$$x^2(2x - 3)(x + 2)(4x - 7).$$

105. Divide numerator by x ; then

$$\begin{array}{r}
 12x^3 - 3x^2 - 8x + 2 \quad 3x^4 - 3x^3 - 8x^2 + 2x + 4 \\
 \hline
 12x^4 - 12x^3 - 32x^2 + 8x + 16 \quad (x + 8) \\
 \hline
 12x^4 - 3x^3 - 8x^2 + 2x \\
 \hline
 - 9x^3 - 24x^2 + 6x + 16 \\
 96x^3 - 24x^2 - 64x + 16 \\
 \hline
 - 35x \quad - 105x^3 \quad 70x \\
 \hline
 3x^3 \quad - 2 \quad 12x^3 - 3x^2 - 8x + 2 \quad (4x - 1) \\
 \hline
 12x^3 \quad - 8x \\
 \hline
 - 3x^2 \quad + 2 \\
 - 3x^2 \quad + 2
 \end{array}$$

G. C. M. is $3x^2 - 2$; dividing numerator and denominator by it, we obtain

$$\frac{x(4x-1)}{x^2-x-2}.$$

106.

$$\begin{array}{r}
 80.00(5.47722\dots \\
 25 \\
 104 \overline{)500} \\
 \underline{416} \\
 1087 \overline{)8400} \\
 \underline{7609} \\
 10947 \overline{)79100} \\
 \underline{76629} \\
 109542 \overline{)247100} \\
 \underline{219084} \\
 1095442 \overline{)2801800} \\
 \underline{2190884}
 \end{array}$$

107.

(i) $12x + 42 - 48x = 45x + 15 - 783$;

$12x - 48x - 45x = 15 - 783 - 42$;

$-81x = -810$, or $x = 10$.

(ii) $35x + 315 - 84x + 168 = 420 - 40x$;

$35x - 84x + 40x = 420 - 315 - 168$;

$-9x = -63$, or $x = 7$.

(iii)

$14x + 9y = 156$,

but from (2)

$14x + 4y = 116$;

$\therefore 5y = 40$, or $y = 8$;

substitute in (2), then $7x + 16 = 53$, or $x = 6$.

108. From (1) $15x - 6y - 9z = 3$,
 from (2) $x - 6y + 4z = 9$;
 $\therefore 14x - 13z = -6$ (A).

Again, from (1), $5x - 2y - 3z = 1$,
 from (2), $14x + 2y - 10z = 28$,
 $\therefore 19x - 13z = 29$ (B).

Subtract (A) from (B), then $5x = 35$, or $x = 7$;
 substitute in (A), then $98 - 13z = -6$, or $z = 8$;
 substitute in (1), then $85 - 2y - 24 = 1$, or $y = 5$.

109. Let x be C's contribution in £s,
 then $x + 15$ is B's;
 and $2x + 30$ is A's;
 $\therefore x + x + 15 + 2x + 30 = 125$;
 $\therefore 4x = 80$, or $x = 20$;

hence C gave £20, B £35, A £70.

110. $x^2 + 16x + 64 = 169$;
 $x + 8 = \pm 13$; $\therefore x = 5$, or -21 .

111. $63 - 49 + 9 - 1 = 22$;
 $3(7 + 9 - 7) = 3 \times 9 = 27$;

hence the difference is 5.

112.
$$\frac{3x^3 - 8x^2y - xy^3 + y^3}{2x^3 - 8x^2y + 13xy^2 - y^3} = \frac{x^2}{-14xy^2 + 2y^3}$$

113.
$$\begin{array}{r} 3x^3 - 7x^2 - 1 \\ 3x^3 + 7x + 5 \\ \hline 9x^5 - 21x^4 - 3x^2 \\ - 21x^4 - 49x^3 - 7x \\ \hline 15x^3 - 35x^2 - 5 \\ \hline 9x^5 - 34x^3 - 38x^2 - 7x - 5 \end{array}$$

114.
$$\begin{array}{r} 3a^2 + 7a + 5 \quad 9a^5 \quad - 31a^3 - 22a^2 + 19a + 10(3a^3 - 7a^2 + a + 2) \\ 9a^5 + 21a^4 + 15a^3 \\ \hline - 21a^4 - 46a^3 - 22a^2 \\ - 21a^4 - 49a^3 - 35a^2 \\ \hline 3a^3 + 13a^2 + 19a \\ 3a^3 + 7a^2 + 5a \\ \hline 6a^2 + 14a + 10 \\ 6a^2 + 14a + 10 \end{array}$$

$$115. \quad \begin{array}{r} a^6 - 4a^5 + 10a^4 - 20a^3 + 25a^2 - 24a + 16 \overline{) a^6 - 2a^5} \\ \underline{4a^5 + 10a^4} \\ 2a^3 - 4a^2 + 3a \end{array}$$

$$\begin{array}{r} \overline{4a^5 + 10a^4} \\ \underline{- 4a^5 + 4a^4} \\ 6a^4 - 12a^3 + 9a^2 \\ \underline{6a^4 - 20a^3 + 25a^2} \\ 8a^3 - 16a^2 - 24a + 16 \\ \underline{- 8a^3 + 16a^2 - 24a + 16} \\ 0 \end{array}$$

$$116. \quad \begin{array}{l} 3x - 12 + 7x = 18 + 4x \\ 6x = 30, \text{ or } x = 5. \end{array}$$

$$117. \quad \begin{array}{l} 28x + 6 = 130 - 3x; \\ 31x = 124, \text{ or } x = 4. \end{array}$$

118. Divide denominator by x ; then

$$\begin{array}{r} 6x^3 + 8x^2 - 2x - 1 \overline{) 12x^3 - 9x^2 - 4x + 3} \\ \underline{12x^3 + 6x^2 - 4x - 2} \\ -5x^2 - 15x^2 + 5 \\ \underline{3x^2} \overline{) 6x^3 + 8x^2 - 2x - 1} \\ \underline{6x^3} \\ 3x^2 - 1 \\ \underline{3x^2} \\ -1 \end{array}$$

Dividing numerator and denominator by $3x^2 - 1$, we obtain $\frac{2x+1}{x(4x-8)}$.

$$119. \quad \begin{array}{r} .0016 \overline{) .04} \\ \underline{.16} \\ .04 \end{array}$$

$$120. \quad x^2 + 15x + \frac{225}{4} = -26 + \frac{225}{4} = \frac{121}{4};$$

$$x + \frac{15}{2} = \pm \frac{11}{2};$$

$$x = \pm \frac{11}{2} - \frac{15}{2} = -2 \text{ or } -13.$$

$$121. \quad \begin{array}{r} 5a + 7b - 3c + d \\ -11a - 5b + 2c + 8d \\ \underline{7a - 2b + 3c - 10d} \\ a + 2c - d \end{array}$$

$$122. \quad \begin{array}{r} 6x^2 + 3y^2 - 4z^2 - 12yz + xz \\ 5x^2 - 3xy + 2y^2 - 11yz + xz \\ \underline{x^2 + 3xy + y^2 - 4z^2 - yz} \end{array}$$

123.

$$\begin{array}{r}
 a^2 + b^2 - c^2 \\
 a^2 - b^2 + c^2 \\
 \hline
 a^4 + a^2b^2 - a^2c^2 \\
 - a^2b^2 - b^4 + b^2c^2 \\
 + a^2c^2 + b^2c^2 - c^4 \\
 \hline
 a^4 - b^4 + 2b^2c^2 - c^4
 \end{array}$$

124.

$$\begin{array}{r}
 x^2 - 6x + 9) x^4 - 12x^3 + 50x^2 - 84x + 45 (x^2 - 6x + 5 \\
 \underline{x^4 - 6x^3 + 9x^2} \\
 - 6x^3 + 41x^2 - 84x \\
 \underline{- 6x^3 + 36x^2 - 54x} \\
 5x^2 - 30x + 45 \\
 \underline{5x^2 - 30x + 45} \\
 0
 \end{array}$$

125.

$$\begin{aligned}
 6x + 6 + 3x + 9 &= 192 - 4x - 8, \\
 6x + 3x + 4x &= 192 - 8 - 6 - 9, \\
 13x &= 169; \therefore x = 13.
 \end{aligned}$$

126.

$$\begin{aligned}
 3x + 5y &= 6, \\
 2x - z &= -3, \\
 3y + 2z &= 14.
 \end{aligned}$$

Multiply equation (2) by 2, and add equation (3);
then $4x + 3y = 8$;

Multiply this by 3; then $12x + 9y = 24$,
but from (1), $12x + 20y = 24$;
 $\therefore -11y = 0$, or $y = 0$;

substitute in (1); then $3x = 6$, or $x = 2$;

substitute in (3); then $2z = 14$, or $z = 7$.

127.

$$\begin{aligned}
 x^2 + \frac{7x}{2} &= \frac{39}{2}, \\
 x^2 + \frac{7x}{2} + \frac{49}{16} &= \frac{39}{2} + \frac{49}{16} = \frac{361}{16}, \\
 x + \frac{7}{4} &= \pm \frac{19}{4}; \therefore x = \pm \frac{19}{4} - \frac{7}{4} = 3 \text{ or } -6\frac{1}{4}.
 \end{aligned}$$

128.

$$\begin{array}{r}
 2x^2 + x - 3) 2x^2 - 9x + 7 (1 \\
 \underline{2x^2 + \quad x - 3} \\
 -10) -10x + 10 \\
 \underline{-10x - 10} \\
 20x - 20 \\
 \underline{20x - 20} \\
 0
 \end{array}$$

Hence L. C. M. of two expressions is $(x-1)(2x+3)(2x-7)$,
or $4x^3 - 12x^2 - 13x + 21$.

Now take this with the third expression:

$$\begin{array}{r}
 3x^3 - 4x + 1 \quad 4x^3 - 12x^2 - 13x + 21 \\
 \quad \quad \quad 8 \\
 \hline
 12x^3 - 36x^2 - 39x + 63 \quad 4x + 63 \\
 12x^3 - 16x^2 + 4x \\
 \hline
 - 20x^2 - 43x + 65 \\
 + 189x^2 - 252x + 63 \\
 - 209x - 209x^2 + 209x \\
 \hline
 x - 1 3x^2 - 4x + 1 (3x - 1) \\
 3x^2 - 3x \\
 - x + 1 \\
 - x + 1
 \end{array}$$

Hence L. C. M. is $(x-1)(3x-1)(4x^2-8x-21)$.

$$\begin{aligned}
 129. \quad & 5x - 3[2x + 9y - 2\{3x - 4y + 4x\}] \\
 & = 5x - 3[2x + 9y - 6x + 8y - 8x] \\
 & = 5x - 6x - 27y + 18x - 24y + 24x \\
 & = 41x - 51y.
 \end{aligned}$$

$$\begin{array}{r}
 130. \quad 4x^3 - 17x + 13 \quad 6x^3 - 17x + 11 \\
 \quad \quad \quad 2 \\
 \hline
 12x^3 - 34x + 22 \quad 3 \\
 12x^3 - 51x + 39 \\
 \hline
 17 17x - 17 \\
 x - 1 4x^3 - 17x + 13 (4x^3 + 4x - 13) \\
 4x^3 - 4x^2 \\
 4x^2 - 17x \\
 4x^2 - 4x \\
 - 13x + 13 \\
 - 13x + 13
 \end{array}$$

Dividing numerator and denominator by $x-1$, we obtain $\frac{4x^2+4x-13}{6x^2+6x-11}$.

$$\begin{array}{r}
 131. \quad \begin{array}{r} 5a^3 - 3a + 2 \\ - 5a^2 + 3a + 7 \\ - 4a^3 + 4a^2 + 3 \\ \hline a^3 - a^2 + 12 \end{array} \quad \begin{array}{r} 2a^3 - 2a^2 - a + 12 \\ a^3 - a^2 + 12 \\ \hline a^3 - a^2 - a \end{array}
 \end{array}$$

$$\begin{array}{r}
 132. \quad \begin{array}{r} a^5 - 3a^3x^2 - 6x^5 \\ 2a^3 + 3a^2x - 4x^3 \\ 2a^5 - 6a^3x^2 - 12a^2x^5 \\ 3a^7x - 9a^5x^3 - 18a^2x^6 \\ - 4a^5x^3 + 12a^2x^5 + 24x^8 \\ \hline 2a^8 + 3a^7x - 6a^6x^2 - 13a^5x^3 - 18a^2x^6 + 24x^8 \end{array}
 \end{array}$$

$$133. \quad \begin{array}{r} 3a - b^2) 243a^5 - b^{10} \quad (81a^4 + 27a^2b^2 + 9a^2b^4 + 3ab^6 + b^8 \\ \underline{243a^5 - 81a^4b^2} \end{array}$$

$$\begin{array}{r} 81a^4b^2 - b^{10} \\ \underline{81a^4b^2 - 27a^2b^4} \\ 27a^2b^4 - b^{10} \\ \underline{27a^2b^4 - 9a^2b^6} \\ 9a^2b^6 - b^{10} \\ \underline{9a^2b^6 - 3ab^8} \\ 3ab^8 - b^{10} \\ \underline{3ab^8 - b^{10}} \end{array}$$

$$134. \quad \begin{array}{r} 1 + 4x + 8x^2 + 6x^3 - 4x^5 + x^6 \quad (1 + 2x + 2x^2 - x^3 \\ \underline{1} \end{array}$$

$$\begin{array}{r} 2 + 2x \quad \underline{4x + 8x^2} \\ 4x + 4x^3 \\ \underline{2 + 4x + 2x^2} \quad 4x^2 + 6x^3 \\ 4x^2 + 8x^3 + 4x^4 \\ \underline{2 + 4x + 4x^2 - x^3} \quad -2x^3 - 4x^4 - 4x^5 + x^6 \\ -2x^3 - 4x^4 - 4x^5 + x^6 \end{array}$$

$$135. \quad \sqrt{1.361} = \sqrt{\frac{1225}{900}} = \frac{35}{30} = 1.1\bar{6}.$$

$$136. \quad \begin{array}{r} x^3 - 4x^2 + 9x - 10) x^3 + 2x^2 - 8x + 20 \quad (1 \\ \underline{x^3 - 4x^2 + 9x - 10} \\ 6) 6x^2 - 12x + 30 \\ \underline{x^2 - 2x + 5} \quad x^3 - 4x^2 + 9x - 10 \quad (x - 2 \\ \underline{x^3 - 2x^2 + 5x} \\ -2x^2 + 4x - 10 \\ \underline{-2x^2 + 4x - 10} \end{array}$$

$x^2 - 2x + 5$ is the g. c. m. of these two expressions; also 2 is g. c. m. of 4 and 6; hence the answer is $2(x^2 - 2x + 5)$.

$$137. \quad \begin{array}{r} .06'30'02'50'60'09 \quad (.251003 \\ 4 \end{array}$$

$$\begin{array}{r} 45 \overline{) 230} \\ \underline{225} \\ 501 \overline{) 502} \\ \underline{501} \\ 502003 \overline{) 1506009} \\ \underline{1506009} \end{array}$$

$$138. \quad \begin{array}{l} 21x + 4y = 185, \\ 20x - 3y = 6; \\ 63x + 12y = 405, \\ 80x - 12y = 24; \\ \therefore 143x = 429, \text{ or } x = 3, \\ \text{substitute in (1), then} \quad 63 + 4y = 185, \text{ or } y = 18. \end{array}$$

139. From (1),
from (2),
∴ by subtraction
but from (3),

$$\begin{aligned}16x + 4y - 12z &= 60, \\21x + 9y - 12z &= 105; \\5x + 5y &= 45, \\x - 5y &= -15; \\\therefore 6x &= 30, \text{ or } x = 5, \\-5y &= -20, \text{ or } y = 4; \\20 + 4 - 3z &= 15, \\\therefore -3z &= 15 - 20 - 4 = -9, \text{ or } z = 3.\end{aligned}$$

substitute in (3), then
substitute in (1), then

$$\begin{aligned}140. \quad x^2 - \frac{8x}{5} &= \frac{21}{5}; \\x^2 - \frac{8x}{5} + \frac{16}{25} &= \frac{21}{5} + \frac{16}{25} = \frac{121}{25}; \\x - \frac{4}{5} &= \pm \frac{11}{5}; \therefore x = 3, \text{ or } -1\frac{1}{5}.\end{aligned}$$

$$141. \quad 27 - 12 + 0 + 60 = 75.$$

$$\begin{array}{r}142. \quad 2x + 3y \quad 32x^5 + 243y^5 \quad (16x^4 - 24x^3y + 36x^2y^2 - 54xy^3 + 81y^4) \\ \quad \quad \quad \underline{32x^5 + 48x^4y} \\ \quad \quad \quad \quad - 48x^4y + 243y^5 \\ \quad \quad \quad \quad \underline{- 48x^4y - 72x^3y^2} \\ \quad \quad \quad \quad \quad 72x^3y^2 + 243y^5 \\ \quad \quad \quad \quad \quad \underline{72x^3y^2 + 108x^2y^3} \\ \quad \quad \quad \quad \quad \quad - 108x^2y^3 + 243y^5 \\ \quad \quad \quad \quad \quad \quad \underline{- 108x^2y^3 - 162xy^4} \\ \quad \quad \quad \quad \quad \quad \quad 162xy^4 + 243y^5 \\ \quad \quad \quad \quad \quad \quad \quad \underline{162xy^4 + 243y^5}\end{array}$$

$$\begin{array}{r}143. \quad \frac{16a^{12} - 24a^{10}b + 25a^8b^2 - 12a^6b^3 + 4a^4b^4}{16a^{12}} \quad (4a^6 - 3a^4b + 2a^2b^2) \\ \quad \quad \quad \underline{8a^6 - 3a^4b} \\ \quad \quad \quad \quad - 24a^{10}b + 25a^8b^2 \\ \quad \quad \quad \quad \underline{- 24a^{10}b + 9a^8b^2} \\ \quad \quad \quad \quad \quad 16a^8b^2 - 12a^6b^3 + 4a^4b^4 \\ \quad \quad \quad \quad \quad \underline{16a^8b^2 - 12a^6b^3 + 4a^4b^4}\end{array}$$

$$144. \quad \sqrt{.00537} = \sqrt{\frac{484}{90000}} = \frac{22}{300} = .07\bar{3}.$$

$$\begin{aligned}145. \quad (i) \quad 3x - 9 - 2x + 4 + x - 1 &= x + 3 + 2x + 4 + 3x + 3; \\3x - 2x + x - x - 2x - 3x &= 3 + 4 + 3 + 9 - 4 + 1; \\-4x &= 16; \text{ or } x = -4. \\(ii) \quad 44x + 154 - 126x + 112 &= 77x - 847; \\44x - 126x - 77x &= -847 - 154 - 112; \\-159x &= -1113; \text{ or } x = 7.\end{aligned}$$

146. $10x + 5y - 20 = 8 + 4x$, or $6x + 5y = 28$,
 $3 - 4x + 18 = 15y - 21$, or $4x + 15y = 42$;
 from (1), $18x + 15y = 84$,
 from (2), $4x + 15y = 42$,
 $14x = 42$, or $x = 3$;
 substitute in (1), then $18 + 5y = 28$, or $y = 2$.

147. From (3), $6x + 4y - 2z = -2$,
 from (2), $y + 2z = 2$;
 \therefore by addition $6x + 5y = 0$;
 From (1), $6x + 12y = 42$,
 $\therefore 7y = 42$, or $y = 6$;
 substitute in (1); then $x + 12 = 7$, or $x = -5$;
 substitute in (2); then $2z = 2 - 6$, or $z = -2$.

148. $x^2 + 25x + \frac{625}{4} = \frac{625}{4} - 100 = \frac{225}{4}$;
 $x + \frac{25}{2} = \pm \frac{15}{2}$; $\therefore x = -5$, or -20 .

149. Let x be one part, and $52 - x$ the other;
 then $x = 2(52 - x) + 4$,
 $3x = 108$, or $x = 36$,
 hence 36 and 16 are the parts.

150. Let x feet be A 's stride, and y feet B 's:
 $\therefore 4x - 3y = 2$;
 also $7x - 6y = 0$;
 hence $x = 4$.

151. $\frac{0+8}{2} - \frac{1+8}{4} + \frac{0+32}{12}$
 $= 4 - 2\frac{1}{4} + 2\frac{2}{3} = 4\frac{5}{12}$.

152.
$$\begin{array}{r} (x-y+z)x^2 - y^2 - z^2 + 2yz(x+y-z) \\ \hline x^3 - xy + xz \\ \hline xy - xz - y^2 - z^2 + 2yz \\ \hline xy \quad - y^2 \quad + yz \\ \hline -xz \quad - z^2 + yz \\ \hline -xz \quad - z^2 + yz \end{array}$$

153.

$$\begin{array}{r}
 2x - 5y^2 \\
 2x - 5y^2 \\
 \hline
 4x^2 - 10xy^2 \\
 - 10xy^2 + 25y^4 \\
 \hline
 4x^2 - 20xy^2 + 25y^4 \\
 2x - 5y^2 \\
 \hline
 8x^3 - 40x^2y^2 + 50xy^4 \\
 - 20x^2y^2 + 100xy^4 - 125y^6 \\
 \hline
 8x^3 - 60x^2y^2 + 150xy^4 - 125y^6
 \end{array}$$

154.

$$\begin{aligned}
 2x + 2x + 3y &= 48, \text{ or } 4x + 3y = 48, \\
 7y - 3x - 2y &= 22, \text{ or } -3x + 5y = 22;
 \end{aligned}$$

from (1),

$$12x + 9y = 144,$$

from (2),

$$-12x + 20y = 88;$$

$$\therefore 29y = 232, \text{ or } y = 8;$$

substitute in (1), then

$$4x + 24 = 48, \text{ or } x = 6.$$

155. Multiply (1) by 6, then $18x + 12y - 6z = 120$,
and from (2), $2x + 3y + 6z = 50$;

$$\therefore 20x + 15y = 170, \text{ or } 4x + 3y = 34 \dots\dots\dots (A).$$

Also subtract (3) from (2), then $x + 4y = 15 \dots\dots\dots (B).$

Multiply (B) by 4; $4x + 16y = 60$,

but from (A);

$$4x + 3y = 34:$$

$$\therefore 13y = 26, \text{ or } y = 2:$$

substitute in (B), then

$$x + 8 = 15, \text{ or } x = 7;$$

substitute in (1), then

$$21 + 4 - z = 20, \text{ or } z = 5.$$

156,

$$\begin{array}{r}
 6x^2 + 7x - 3 \quad 12x^2 + 16x - 3 \quad (2) \\
 12x^2 + 14x - 6 \\
 \hline
 2x + 3 \quad 6x^2 + 7x - 3 \quad (3x - 1) \\
 6x^2 + 9x \\
 \hline
 -2x - 3 \\
 -2x - 3
 \end{array}$$

Dividing both expressions by $2x + 3$, we find as L. C. M.

$$(2x + 3)(3x - 1)(6x - 1).$$

157.

$$\begin{aligned}
 2a - 2b - \{a + [b - 2a + 2b]\} \\
 = 2a - 2b - \{a + b - 2a + 2b\} \\
 = 2a - 2b - a - b + 2a - 2b = 3a - 5b.
 \end{aligned}$$

158. Let x be one part, and $150 - x$ the other:

$$\text{then } \frac{x}{23} + \frac{150 - x}{27} = 6;$$

$$27x + 8450 - 23x = 3726;$$

$$4x = 276; \text{ or } x = 69;$$

hence 69 and 81 are the parts.

159. Let x and y be the digits, so that $10x + y$ is the number;
then $x + y = 13$,

and $10x + y + 9 = 10y + x$;

from (2) $9x - 9y = -9$, or $x - y = -1$;

hence by addition $2x = 12$, or $x = 6$,

hence $y = 7$; and the number is 67.

160. $(x - y + z)^2 = x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$,
hence the difference is $2xy - 2xz + 2yz$.

$$\begin{array}{r}
 161. \quad a + 2b - c) a^3 + 8b^3 - c^3 + 6abc(a^3 - 2ab + ac + 4b^3 + 2bc + c^3) \\
 \underline{a^3 + 2a^2b - a^2c} \\
 -2a^2b + a^2c + 6abc + 8b^3 - c^3 \\
 \underline{-2a^2b - 4ab^2 + 2abc} \\
 a^2c + 4ab^2 + 4abc + 8b^3 - c^3 \\
 \underline{a^2c + 2abc - ac^2} \\
 4ab^2 + 2abc + ac^2 + 8b^3 - c^3 \\
 \underline{4ab^3} \\
 2abc + ac^2 + 4b^2c - c^3 \\
 \underline{2abc} \\
 ac^2 + 2bc^2 - c^3 \\
 \underline{ac^2 + 2bc^2 - c^3} \\
 0
 \end{array}$$

$$\begin{array}{r}
 162. \quad \begin{array}{r} 2'91'04'36(1706 \\ 1 \\ 27 \overline{) 191} \\ \underline{189} \\ 3406 \overline{) 20436} \\ \underline{20436} \end{array}
 \end{array}$$

$$\begin{array}{r}
 163. \quad \begin{array}{r} x^4 - 4ax^3 + 6a^2x^2 - 4a^3x + a^4(x^2 - 2ax + a^2) \\ x^4 \\ \hline 2x^2 - 2ax \overline{) -4ax^3 + 6a^2x^2} \\ \underline{-4ax^3 + 4a^2x^2} \\ 2x^2 - 4ax + a^2 \overline{) 2a^2x^2 - 4a^3x + a^4} \\ \underline{2a^2x^2 - 4a^3x + a^4} \\ x^2 - 2ax + a^2(x - a) \\ x^2 \\ \hline 2x - a \overline{) -2ax + a^2} \\ \underline{-2ax + a^2} \\ 0 \end{array}
 \end{array}$$

$$\begin{array}{r}
 164. \quad \begin{array}{l} a - [b - \{2a - 3b + 3a + c\}] \\ = a - [b - 2a + 3b - 3a - c] \\ = a - b + 2a - 3b + 3a + c \\ = 6a - 4b + c. \end{array}
 \end{array}$$

165. (i) $18x - 27 - 14x - 84 = 21x - 315;$
 $-17x = -204, \text{ or } x = 12.$

(ii) $5x - 6y = 0,$
 $9x - 3 - 8y + 2 - 6 = 0; \text{ or } 9x - 8y = 7;$
 from (1), $20x - 24y = 0,$
 from (2), $27x - 24y = 21;$
 $\therefore 7x = 21, \text{ or } x = 3;$
 substitute in (1), then $15 - 6y = 0, \text{ or } y = 2\frac{1}{2}.$

166. From (1), $6x - 4y = 4,$
 from (2), $4y - 5z = 2;$
 $\therefore 6x - 5z = 6;$
 but from (3), $6x + 9z = 6;$
 $\therefore 14z = 0, \text{ or } z = 0;$
 substitute in (2), then $4y - 0 = 2, \text{ or } y = \frac{1}{2};$
 substitute in (3), then $2x + 0 = 2; \text{ or } x = 1.$

167. $\frac{12x^2(x+5)(x+1)}{32x^3(x+5)(x-1)} = \frac{3(x+1)}{8x(x-1)}.$

168. Divide the first by x , and the second by a^2 :

$$\begin{array}{r} x^3 - 2x^2 - 3x + 6 \quad x^3 - 2x^2 - 5x + 10 \quad (1) \\ \underline{x^3 - 2x^2 - 3x + 6} \\ -2 \quad -2x + 4 \\ \underline{x-2} \quad x^3 - 2x^2 - 3x + 6 \quad (x^2 - 3) \\ \underline{x^3 - 2x^2} \\ -3x + 6 \\ \underline{-3x + 6} \end{array}$$

G. C. M. is $a^2x(x-2)$, L. C. M. is $a^2x(x-2)(x^2-3)(x^2-5).$

169. $x^2 - \frac{12x}{5} + \frac{36}{25} = \frac{36}{25} - \frac{4}{5} = \frac{16}{25};$
 $x - \frac{6}{5} = \pm \frac{4}{5};$
 $x = 2 \text{ or } \frac{2}{5}.$

170. Let x be the number of florins,
 and y the number of half-crowns;

then $2x + \frac{5y}{2} = 67,$

also $x + \frac{5y}{2} = 56;$

$\therefore x = 11, \text{ and } y = 18.$

$$171. \quad (i) \quad 27 - 36 + 9 - 0 + 1 = 1;$$

$$(ii) \quad \frac{8}{4} - \frac{6}{3} + \frac{4}{2} = 2 - 2 + 2 = 2.$$

$$172. \quad \begin{array}{r} 13x^3 - 14xy - y^3 \\ - 9x^2 + 5xy - y^3 \\ - 6x^2 - 15xy + 11y^3 \\ \hline 10x^3 + 24xy - 3y^3 \\ 8x^3 \quad + \quad 6y^3 \end{array}$$

$$173. \quad \begin{array}{r} x^3 - 10x^2 + 17x + 12 \quad) \quad 2x^3 - 19x^2 + 27x + 20 \quad (2 \\ \underline{2x^3 - 20x^2 + 34x + 24} \\ x^2 - 7x - 4 \quad) \quad x^3 - 10x^2 + 17x + 12 \quad (x - 3 \\ \underline{x^3 - 7x^2 - 4x} \\ - 3x^2 + 21x + 12 \\ \underline{- 3x^2 + 21x + 12} \end{array}$$

Also 2 is G. C. M. of 6 and 4x; hence the complete G. C. M. is $2(x^2 - 7x - 4)$.

$$174. \quad (i) \quad \begin{array}{l} 2x - 6 - 5x + 20 = x - 6, \\ 2x - 5x - x = -6 + 6 - 20, \\ \therefore -4x = -20, \text{ or } x = 5. \end{array}$$

$$(ii) \quad \begin{array}{l} 3x - 2x + 28 = 60 - 3x, \\ \therefore 4x = 32, \text{ or } x = 8. \end{array}$$

$$175. \quad \begin{array}{r} x^4 - 6x^3y + 25x^2y^2 - 48xy^3 + 64y^4 \quad (x^2 - 3xy + 8y^2 \\ \hline 2x^3 - 3xy \quad \begin{array}{l} - 6x^2y + 25x^2y^2 \\ - 6x^2y + 9x^2y^2 \end{array} \\ \hline 2x^3 - 6xy + 8y^3 \quad \begin{array}{l} 16x^2y^3 - 48xy^3 + 64y^4 \\ 16x^2y^3 - 48xy^3 + 64y^4 \end{array} \end{array}$$

176. Let x be the number of minute spaces traversed by the hour-hand; then the minute-hand has traversed

$$40 + x - 15;$$

$$\text{hence} \quad 40 + x - 15 = 12x, \text{ or } x = \frac{25}{11} = 2\frac{3}{11};$$

hence the time is $8:27\frac{3}{11}$.

$$177. \quad \begin{array}{r} 74'68'41 \cdot 64 \quad (864 \cdot 2 \\ 64 \\ 166 \overline{)1068} \\ \underline{996} \\ 1724 \overline{)7241} \\ \underline{6896} \\ 17282 \overline{)34564} \\ \underline{34564} \end{array}$$

178.

$$\begin{array}{r}
 x - 3y \mid x^4 - 81y^4 \quad (x^2 + 8x^2y + 9xy^2 + 27y^3) \\
 \underline{x^4 - 8x^2y} \\
 8x^2y - 81y^4 \\
 \underline{8x^2y - 9x^2y^2} \\
 9x^2y^2 - 81y^4 \\
 \underline{9x^2y^2 - 27xy^3} \\
 27xy^3 - 81y^4 \\
 \underline{27xy^3 - 81y^4} \\
 0
 \end{array}$$

179. (i)

$$35x + 315 - 84x + 168 = 420 - 40x;$$

$$35x - 84x + 40x = 420 - 315 - 168;$$

$$-9x = -63; \text{ or } x = 7.$$

(ii)

$$2x - y + 21x = 14y - 42, \text{ or } 23x - 15y = -42;$$

$$6y + 18 + 5y - 5x = 60x - 240, \text{ or } -65x + 11y = -258;$$

from (1),

$$253x - 165y = -462,$$

from (2),

$$-975x + 165y = -3870;$$

$$\therefore -722x = -4332, \therefore x = 6;$$

substitute in (1), then $138 - 15y = -42$, or $y = 12$.

180. From (2),

$$4x + 6y - 2z = 40,$$

from (1),

$$4x - 5y + 2z = 6,$$

$$\therefore 8x + y = 46 \dots\dots\dots (A);$$

from (2),

$$6x + 9y - 3z = 60,$$

from (3),

$$7x - 4y + 3z = 35,$$

$$\therefore 13x + 5y = 95 \dots\dots\dots (B);$$

but from (A),

$$40x + 5y = 230;$$

$$\therefore 27x = 135, \text{ or } x = 5;$$

substitute in (A); then

$$40 + y = 46, \text{ or } y = 6;$$

substitute in (2); then

$$10 + 18 - z = 20, \text{ or } z = 8.$$

181.

$$\begin{array}{r}
 x - y + z \mid x^3 - 2xy + y^3 - x^2(x - y - z) \\
 \underline{x^3 - xy + xz} \\
 -xy - xz + y^3 - x^3 \\
 \underline{-xy + y^3 - yz} \\
 -xz + yz - x^3 \\
 \underline{-xz + yz - x^3} \\
 0
 \end{array}$$

182.

$$a^2 + 2ab + b^2 + b^3 - (a^2 - 2ab + b^2 + b^3) = 4ab.$$

183. (i)

$$\frac{2}{2} - \frac{1}{8} = \frac{7}{8},$$

(ii)

$$\frac{2}{-\frac{12}{7}} - \frac{1}{\frac{25}{49} - 1} = -\frac{7}{6} + \frac{49}{24} = \frac{21}{24} = \frac{7}{8}.$$

184. $20x - 80 - 80x + 75 = 36x - 132 - 195;$
 $-46x = -322, \text{ or } x = 7.$

185. $10x - 25 - 9y + 3 = 60 - 10y - 45; \text{ or } 10x + y = 37,$
 $x + 2y = 17,$

but from (1), $20x + 2y = 74;$
 $\therefore 19x = 57, \text{ or } x = 3;$

substitute in (2), then $2y = 14, \text{ or } y = 7.$

186.
$$\begin{array}{r} a^3 + 4b^3 + 9c^3 + 3ab + 3ac - 6bc \\ a - 2b - 3c \\ \hline a^3 + 4ab^2 + 9ac^2 + 2a^2b + 3a^2c - 6abc \\ - 4ab^2 - 2a^2b - 6abc - 8b^3 - 18bc^2 + 12b^2c \\ - 9ac^2 - 3a^2c - 6abc - 27c^3 + 18bc^2 - 12b^2c \\ \hline a^3 - 18abc - 8b^3 - 27c^3 \end{array}$$

187.
$$\begin{array}{r} x^2 - 2xy + y^2 \quad x^4 - 4xy^3 + 3y^4 \quad (x^2 + 2xy + 3y^2) \\ x^4 - 2x^2y + x^2y^2 \\ \hline 2x^2y - x^2y^3 - 4xy^3 + 3y^4 \\ 2x^2y - 4x^2y^2 + 2xy^3 \\ \hline 3x^2y^3 - 6xy^3 + 3y^4 \\ 3x^2y^3 - 6xy^3 + 3y^4 \end{array}$$

188. Let x £s be price of a cow,
 and y £s sheep;

then $12x + 20y = 335,$

also $10(x + 2) + 26\left(y + \frac{3}{4}\right) = 385;$

from (2) $10x + 26y = 335 - 20 - 19\frac{1}{2} = 295\frac{1}{2};$
 $\therefore 60x + 156y = 1773,$

and from (1) $60x + 100y = 1675;$
 $\therefore 56y = 98, \text{ or } y = 1\frac{3}{4};$

substitute in (1), then $12x + 35 = 335, \text{ or } x = 25.$

189.
$$\begin{array}{r} x^2 + x - 2 \quad x^3 - 1 \quad (x - 1) \\ x^3 + x^2 - 2x \\ \hline -x^2 + 2x - 1 \\ -x^2 - x + 2 \\ \hline 3) 3x - 3 \\ x - 1 \quad x^2 + x - 2 \quad (x + 2) \\ x^2 - x \\ \hline 2x - 2 \\ 2x - 2 \end{array}$$

Hence L. C. M. is

$(x - 1)(x + 2)(x^2 + x + 1).$

190.

$$\begin{aligned}
 6x^3 - 3x &= 4x^3 + 2x - 3, \\
 2x^3 - 5x &= -3, \\
 x^3 - \frac{5x}{2} + \frac{25}{16} &= \frac{25}{16} - \frac{3}{2} = \frac{1}{16}, \\
 x - \frac{5}{4} &= \pm \frac{1}{4}; \therefore x = 1\frac{1}{2}, \text{ or } 1.
 \end{aligned}$$

191.

$$81 - 12 = 69; x^3 + 3x^2 = 4x^3.$$

192.

$$\begin{array}{r}
 a^2 + 2ab + 2b^2 \bigg| a^4 + 4b^4 \quad (a^2 - 2ab + 2b^2) \\
 \underline{a^4 + 2a^2b + 2a^2b^2} \\
 -2a^2b - 2a^2b^2 + 4b^4 \\
 \underline{-2a^2b - 4a^2b^2 - 4ab^3} \\
 2a^2b^2 + 4ab^3 + 4b^4 \\
 \underline{2a^2b^2 + 4ab^3 + 4b^4} \\
 0
 \end{array}$$

193.

$$\begin{array}{r}
 .00'18'17'99'90'44 \quad (.042638) \\
 16 \\
 82 \overline{) 217} \\
 \underline{164} \\
 846 \overline{) 5399} \\
 \underline{5076} \\
 8523 \overline{) 82390} \\
 \underline{25569} \\
 85268 \overline{) 682144} \\
 \underline{682144} \\
 0
 \end{array}$$

194.

$$\begin{aligned}
 15a - \{3a - 22b - 10c + 5a - 6c + 3a + 12b\} \\
 = 15a - 3a + 22b + 10c - 5a + 6c - 3a - 12b \\
 = 4a + 10b + 16c.
 \end{aligned}$$

195. (i)

$$\begin{aligned}
 25x - 15 - 25x + 53 + 64x - 88 &= 60x - 40 + 6; \\
 4x &= 16, \text{ or } x = 4.
 \end{aligned}$$

(ii)

$$\begin{aligned}
 2x - y &= 3, \\
 3x - 4y &= 2x, \text{ or } x - 4y = 0, \\
 \text{from (2),} \quad 2x - 8y &= 0, \\
 \text{from (1),} \quad 2x - y &= 3;
 \end{aligned}$$

hence

$$-7y = -3, \text{ or } y = \frac{3}{7};$$

substitute in (1),

$$2x = 3\frac{3}{7}, \text{ or } x = 1\frac{1}{2}.$$

196. From (1), $8x + 4y - 3z = 6$,
 from (2), $3x + 9y - 3z = 21$,
 $\therefore 5x - 5y = -15$, or $x - y = -3$ (A).
 Again, from (2), $4x + 12y - 4z = 28$,
 from (3), $4x - 5y + 4z = 8$,
 $\therefore 8x + 7y = 36$; (B),
 but from (A), $7x - 7y = -21$;
 $\therefore 15x = 15$, or $x = 1$;
 substitute in (A), then $1 - y = -3$, or $y = 4$;
 substitute in (2), then $1 + 12 - z = 7$, or $z = 6$.

197. $x^2 - 10 = \frac{20}{3} - \frac{5x}{3}$;
 $x^2 + \frac{5x}{3} + \frac{25}{36} = \frac{25}{36} + \frac{20}{3} + 10 = \frac{625}{36}$;
 $x + \frac{5}{6} = \pm \frac{25}{6}$; $\therefore x = 3\frac{1}{2}$, or -5 .

198.
$$\begin{array}{r} x^3 - 4x^2 - 2x^2 + 5x - 10(x - 2) \\ x^3 - 4x \\ \hline -2x^2 + 9x - 10 \\ -2x^2 + 8 \\ \hline 9x - 18 \\ 9 \overline{) 9x - 18} \\ x - 2 \end{array}$$

 $(x - 2)x^2 - 4(x + 2)$
 $x^3 - 2x^2$
 $\underline{2x - 4}$
 $2x - 4$

Also $3y$ is G. C. M. of $3y$ and $6y^2$;
 \therefore complete G. C. M. is $3y(x - 2)$.

199. Let x be B's money in shillings,
 then $5x$ is A's;
 $\therefore 5x - 23 = 2(x + 23)$;
 hence $3x = 69$; or $x = 23$, and $5x = 115 = \text{£}5. 15s.$

200. Let x and y be the prices per lb., in shillings:
 then $3x = 4y$; also $9x + 8y = 45$;
 from (1), $9x - 12y = 0$;
 \therefore by subtraction $20y = 45$, or $y = 2\frac{1}{4} = 2s. 8d.$;
 substitute in (1); then $x = \frac{9}{3} = 3s.$

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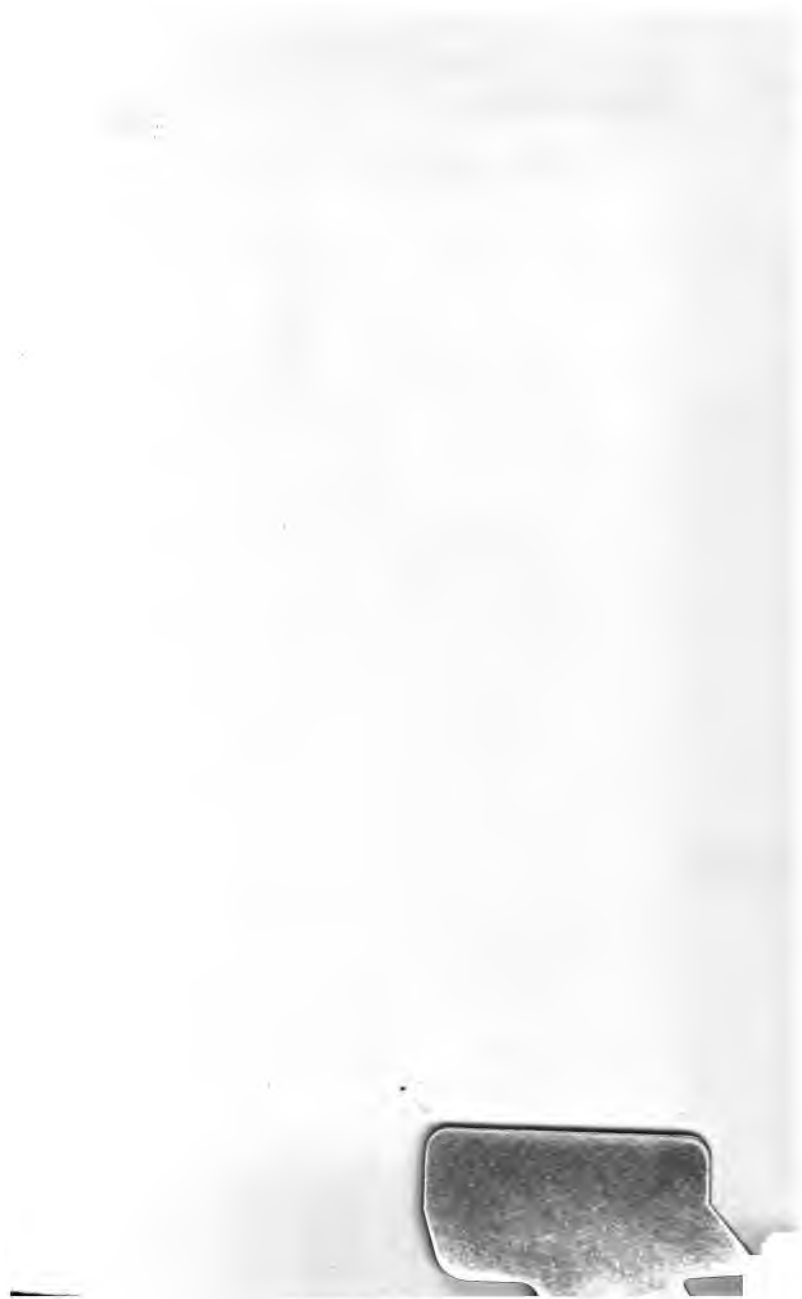
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